
The logo for MEscope features the letters 'ME' in a large, bold, red font. The letter 'E' has a white circle with a red dot in the center, resembling an eye or a sensor. To the right of 'ME', the word 'scope' is written in a dark grey, lowercase, sans-serif font.

MEscope

VES-4000 Modal Analysis

March 7, 2025



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Vibrant Technology, Inc.

13275 East Fremont Place
Suite 200
Centennial, CO 80112 USA

phone: (831) 430-9045

fax: (831) 430-9057

E-mail: support@vibetech.com

<http://www.vibetech.com>

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VES-4000 Modal Analysis

NOTE: If the **VES-4000 Modal Analysis** option is authorized by your MEscape license, the following commands are enabled in the Data Block (**BLK**) and Shape Table (**SHP**) windows. Execute **Help | License Manager** to verify the Options authorized by your MEscape license.

Additional Data Block (BLK) Commands

- Transform | Increase Resolution
- Transform | Window **M#s** (DeConvolution window)
- Curve Fit | Open Curve Fitting
- Curve Fit | Delete All Fit Data
- Curve Fit | Quick Fit
- Curve Fit | Mode Indicator Menu
- Curve Fit | Frequency Damping Menu
- Curve Fit | Residues
- Curve Fit | Modal Parameters Menu
- Curve Fit | Fit Functions Menu
- Curve Fit | Close Curve Fitting

Additional Shape Table (SHP) Commands

- Display | **MAC**
- Display | **SDI**
- Display | **Participation**
- Tools | Synthesize **FRFs**
- Tools | Scaling | **UMM** to Residue Shapes
- Tools | Scaling | Residue to **UMM** Shapes
- Tools | Scaling | Unscaled to Scaled Shapes
- Tools | Scaling | Rapid Test Residues to **UMM** Shapes

Data Block (BLK) Transform Menu

Transform / Increase Resolution

Increases the resolution (reduces the spacing) between samples of data in either time or frequency domain data.

- **FRF**-based curve fitting is improved by increasing the number of samples surrounding each resonance peak
- **Linear interpolation** is used between adjacent samples to create new samples evenly spaced between the original samples
- Each time this command is executed, the number of samples (**Block Size**) is **doubled** in a Data Block (**BLK**).

Transform / Window M#s / (DeConvolution window)

Applies the DeConvolution window to **all** (or **selected**) **M#s** in a Data Block (**BLK**).

The DeConvolution window must be applied to **output-only** measurements before **FRF**-based curve fitting methods can be used on them.

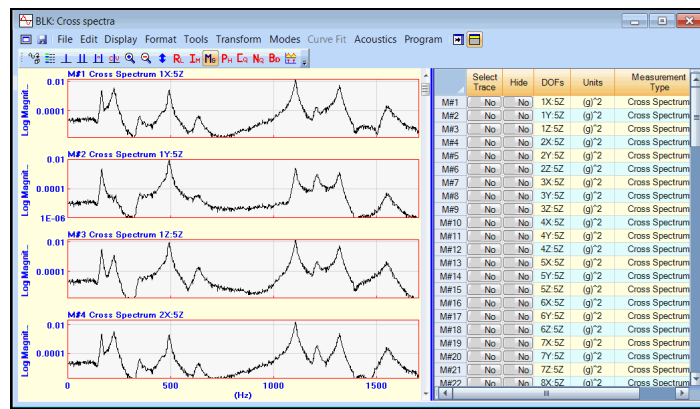
- The DeConvolution window smoothly zeroes (removes) the **second half** of each time waveform

When this command is executed, the following steps are carried out,

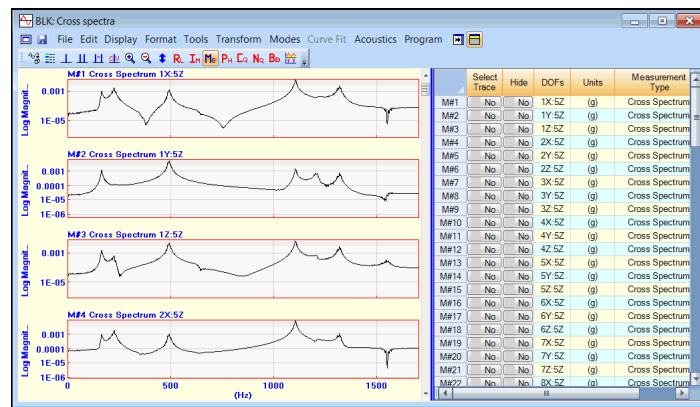
1. **All M#s** are transformed to the time domain.
2. The DeConvolution window is applied to **all** (or **selected**) **M#s**
3. **All M#s** are transformed back to the frequency domain.

The figures below show Cross spectra **before & after** the DeConvolution window was applied to them

Notice that **noise is also removed** from the measurements by the DeConvolution window



Cross Spectra Before Devolution Windowing.



Cross Spectra After Devolution Windowing.

What is a Modal Model?

A Modal Model is a set of mode shapes that has been scaled to preserve the mass, stiffness & damping properties of a structure.

MEscope uses two different Modal Models, **Residue** mode shapes and **UMM** mode shapes.

Residue mode shapes are obtained by curve fitting a set of *calibrated FRFs*.

- **Residue** mode shapes can also be obtained by re-scaling **UMM** mode shapes.

see [Tools](#) | [Scaling](#) | [UMM to Residue Shapes](#).

- **UMM** mode shapes are obtained by re-scaling **Residue** mode shapes.

see [Tools](#) | [Scaling](#) | [Residues to UMM Shapes](#).

- **UMM** mode shapes are also obtained by solving for the modes of an **FEA** model.

Single Reference Modal Test

The most common type of **Experimental Modal Analysis (EMA)** is done using a single reference.

A single reference **EMA** can be performed in two ways,

1. Using a fixed reference response sensor and a roving exciter
2. Using a fixed reference exciter and a roving response sensor

Roving Impact Test

In a roving impact test, a single *fixed reference response* sensor is used, and the structure is excited at multiple DOFs using a *roving* impactor.

- Each **FRF** is calculated between the response signal from the fixed **DOF** and the excitation signal from a different **DOF**.
- The impact hammer applies a broad-band impulsive force to the structure, which excites many resonances at a time
- The excitation force is measured with a load cell attached to the head of the hammer

Roving Response Test

In a roving response test, a single *fixed reference* exciter is used to excite the structure, and a *roving response* sensor is used to measure responses at multiple DOFs.

Each **FRF** is calculated between the excitation signal and the response signal from a different **DOF**.

A reference excitation force can be provided in two ways,

1. Attach a shaker to the structure and drive it with a broad band signal
2. Impact the structure *at the same DOF* throughout the modal test.

Maxwell's Reciprocity

Maxwell's Reciprocity: The structural response at **DOF A** due to an excitation force applied at **DOF B** is *the same* as the structural response at **DOF B** due to the same excitation force applied at **DOF A**.

Maxwell's Reciprocity is assumed to be valid in a single-reference modal test.

- *Any Reference DOF* can be used during a single-reference modal test.
- A Roving Impact test provides mode shapes with a shape component for each **DOF** where the structure is impacted.

- A **Roving Response test** provides mode shapes with a shape component for *each DOF of the response sensor*.
- Using a **tri-axial response sensor** in a **Roving Response test**, provides mode shapes that contain 3D motion of the structure at each response point.

What is FRF-Based Curve Fitting?

FRF-based curve fitting is a process of matching an **FRF parametric model** in a least-squared-error sense to a set of experimental data. It is also referred to as **modal parameter estimation**.

- **FRF-based curve fitting** provides modal parameters (**frequency, damping & mode shape**) for each mode that is identified in the frequency span of the experimental data.
- After curve fitting is completed, mode shapes are stored into a Shape Table (**SHP**) from which they can be displayed in animation on a 3D model of the test article.

Three Curve Fitting Steps

In MEscape, curve fitting is done in three steps.

1. Determine the **Number of Modes** in a frequency band of experimental data
2. Estimate modal **Frequency & Damping** for all modes in the frequency band
3. Estimate modal **Residues** (mode shape components) for *each* mode and *each* measurement function

FRFs in Terms of Modal Parameters

FRF-based curve fitting *assumes that any FRF* calculated between two DOFs of a vibrating structure *can be completely represented in terms of modal parameters*.

Partial Fraction Expansion of the FRF Matrix

The following **parametric model** is used in **FRF**-based curve fitting to estimate modal parameters from experimental **FRF** data.

$$[H(\omega)] = \sum_{k=1}^{\text{modes}} \frac{[R(k)]}{j\omega - p(k)} + \frac{[R^*(k)]}{j\omega - p^*(k)}$$

$[H(\omega)] \rightarrow$ **FRF** matrix (**n** by **n**)

n \rightarrow number of DOFs of the **FRF** matrix

ω \rightarrow frequency variable (in **Hz** or **radians / second**)

$p(k)$ \rightarrow pole for mode(k): $p(k) = -\sigma(k) + j \omega(k)$

$\sigma(k)$ \rightarrow damping decay constant for mode(k) (in Hz or radians / second)

$\omega(k)$ \rightarrow damped natural frequency for mode(k) (in Hz or radians / second)

$[R(k)] \rightarrow$ **Residue** matrix for mode(k) (**n** by **n**)

modes \rightarrow number of modes of interest

* \rightarrow denotes the complex conjugate

j \rightarrow denotes the imaginary axis in the complex plane

- The **FRF** matrix is a *summation of pairs of terms*, each pair containing the contribution of a *single mode*.
- Each pair in the summation consists of a *positive frequency* term and a *complex conjugate negative frequency* term *for each mode*.
- The unknown parameters of the **FRF**-based parametric model are the *modal frequency, damping & Residue* (mode shape component) for each mode of interest in the frequency band of the **FRFs**.
- The residues obtained from curve fitting *one or more FRFs* are saved as a **Residue** mode shape *for each mode*.

Global versus Local Curve Fitting

During global curve fitting, a *global frequency & damping estimate* is obtained from *all FRFs* for each mode.

During local curve fitting, a *local frequency & damping estimate* is obtained from *each FRF* for each mode.

The *denominator* of each term in the **FRF** parametric model contains the *same pole (frequency & damping)* for *each mode*.

- Modal frequency & damping are *global properties* of a structure

Modal Residues

A **Residue** is the "*strength*" of a resonance.

The *larger* its **Residue** value, the *stronger* a resonance is relative to other resonances.

$$\text{Residue units} = (\text{FRF units}) \times (\text{radians / second})$$

Each **Residue** has the *same DOFs* as the **FRF** from which it was obtained by **FRF**-based curve fitting.

FRF-based Curve Fitting

During **FRF**-based curve fitting, *four modal parameters* are estimated *for each mode*.

- The pole location or *frequency & damping* of each mode

$$p(k) = -\sigma(k) + j \omega(k), \quad p(k) - \text{pole of mode}(k)$$
- The complex **Residue** (*magnitude & phase*) for each mode

Frequency & Damping Estimates

Damped Natural Frequency

The frequency estimate *for each mode*(k) is listed as the *damped natural frequency* $\omega(k)$ in the Modal Parameters spreadsheet.

Modal Damping

The damping estimate for each mode(k) is listed as the damping decay constant and the percent of critical damping in the Modal Parameters spreadsheet.

Percent of Critical Damping

Percent of critical damping for mode(k) is defined as,

$$\zeta(k) = \frac{\sigma(k)}{\sqrt{\omega(k)^2 + \sigma(k)^2}} \quad (\%)$$

Damping Decay Constant

The damping decay constant for mode(k) is defined as,

$$\sigma(k) = \frac{\omega(k) \zeta(k)}{\sqrt{1 - \zeta(k)^2}} \quad (\text{Hz})$$

Half-power Point (3 dB bandwidth) Damping

The damping decay constant is *one-half of the width* of the resonance peak.

$$(\omega_2 - \omega_1) = 2\sigma(k) = \Delta\omega_{3\text{dB}}(k) \quad (\text{Hz})$$

- The half-power points are **3 dB less** than the power at the resonance peak
- This half-power point bandwidth estimate of modal damping is also called the **3 dB bandwidth damping**
- The width of the resonance peak in an **FRF** at **70.7%** of the magnitude of the resonance peak is equal to **twice the damping decay constant**.

The wider the resonance peak, the greater the modal damping.

Quality Factor and Loss Factor

The Loss factor is equal to twice the percent of critical damping.

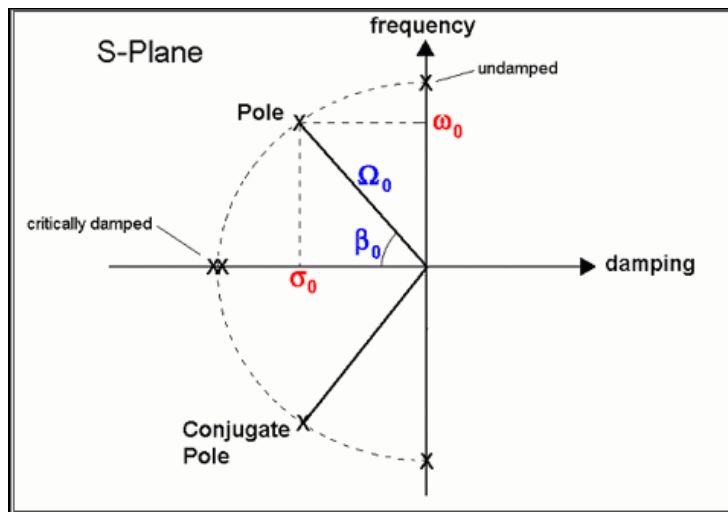
$$\eta(k) = 2\zeta(k)$$

The Quality Factor Q(k) is the *inverse of the Loss factor*.

$$Q(k) = \frac{1}{\eta(k)}$$

S-Plane Plot of Frequency & Damping

The four definitions of modal frequency & damping are plotted on the top view of the S-plane (or complex frequency plane) below.



S-plane Showing Modal Frequency & Damping Terms.

- Modal frequency & damping are defined in several ways,

$\mathbf{p(k)} = -\sigma(\mathbf{k}) + j\omega(\mathbf{k}) \rightarrow$ pole location of mode(k) (Hz)

$\omega(\mathbf{k}) \rightarrow$ damped natural frequency of mode(k) (Hz)

$\sigma(\mathbf{k}) \rightarrow$ damping decay constant (also called 3 dB bandwidth or half-power point damping) of mode(k) (Hz)

$\zeta(\mathbf{k}) \rightarrow$ damping ratio (%)

$\Omega(\mathbf{k}) \rightarrow$ undamped natural frequency of mode(k) (Hz)

$$\Omega(\mathbf{k}) = \sqrt{\omega(\mathbf{k})^2 + \sigma(\mathbf{k})^2} \quad (\text{Hz})$$

Half-power Point Bandwidth (3 dB Bandwidth)	$(\omega_2 - \omega_1) = 2\sigma(\mathbf{k}) = \Delta\omega_{3\text{dB}}(\mathbf{k}) \quad (\text{Hz})$
Damping Ratio (Percent of Critical)	$\zeta(\mathbf{k}) = \frac{\sigma(\mathbf{k})}{\sqrt{\omega(\mathbf{k})^2 + \sigma(\mathbf{k})^2}} \quad (\%)$
Decay Constant	$\sigma(\mathbf{k}) = \frac{\omega(\mathbf{k})\zeta(\mathbf{k})}{\sqrt{1 - \zeta(\mathbf{k})^2}} \quad (\text{Hz})$
Loss Factor	$\eta(\mathbf{k}) = 2\zeta(\mathbf{k})$
Quality Factor	$Q(\mathbf{k}) = \frac{1}{\eta(\mathbf{k})}$

Residues Versus Mode Shapes

The **FRF** matrix can be expressed in terms of modal *poles & mode shapes* as

$$[H(\omega)] = \sum_{k=1}^{\text{modes}} \frac{A(k)\{u(k)\}\{u(k)\}^t}{j\omega - p(k)} + \frac{A^*(k)\{u^*(k)\}\{u^*(k)\}^t}{j\omega - p^*(k)}$$

$p(k) \rightarrow$ pole of mode(k) (Hz)

$p(k) = -\sigma(k) + j\omega(k)$

$A(k) \rightarrow$ scaling constant for mode (k)

$\{u(k)\} \rightarrow$ mode shape for the mode (k) (**n - vector**)

t \rightarrow denotes the transposed vector

Relationship Between Residues & Mode Shapes

The **Residue** matrix $[R(k)]$ for mode(k) is written in terms its mode shape vector $\{u(k)\}$ as,

$$[R(k)] = A(k)\{u(k)\}\{u(k)\}^t \quad k = 1, \dots, \text{modes}$$

- Each mode shape (a **column** vector $\{u(k)\}$, is multiplied by *itself transposed* (a **row** vector $\{u(k)\}^t$)
- This vector product, (a **column** vector times a **row** vector), is called an **outer product**, and it yields a **matrix**
- **Residue** matrix $[R(k)] \rightarrow$ **Outer product of mode shape vectors** multiplied by the **scaling constant** $A(k)$
 - The **Residue** matrix has **unique engineering units**
 - A mode shape is **unique in shape** but has **no engineering units**
 - The scaling constant $A(k)$ makes the **Residue** matrix equal to the **outer product of mode shape vectors**

Mode Shape Scaling

Since scaling of mode shapes is arbitrary, the scaling constant $A(k)$ can be chosen so that $A(k) = 1$.

With $A(k) = 1$, the relationship between the **Residue** and the **mode shape** for each mode(k) becomes,

$$[R(k)] = \{u(k)\}\{u(k)\}^t \quad k = 1, \dots, \text{modes}$$

Fundamental Modal Testing Criterion

The following conclusions can be drawn from the relationship between the **Residue** matrix and mode shapes.

Every row & every column of the **Residue** matrix contains the mode shape multiplied by a different mode shape component

Any row or any column of the **Residue** matrix contains the same mode shape for each mode

Any row or any column of the **FRF** matrix can be curve fit to extract the mode shape for each mode

Mode Shape Node Points

If a row or column of the **Residue** matrix corresponds to a *node point (zero value)* of the mode shape, the *entire row or column* of residues is **zero**.

- *None* of the **FRF** measurements will contain a resonance peak as evidence of the mode.
- A *multi-reference modal test* and *multi-reference curve fitting* might be required to extract all modes

Local Versus Global Modes

Many structures exhibit resonant vibration in a *localized region* of the structure where energy becomes *trapped between stiff boundaries* causing a standing wave of vibration, or *local mode shape*.

Local mode shapes are non-zero in a local region and are zero elsewhere on a structure.

Global mode shapes are non-zero everywhere, except at node points.

Curve Fitting Guidelines

1. Overlay the FRFs

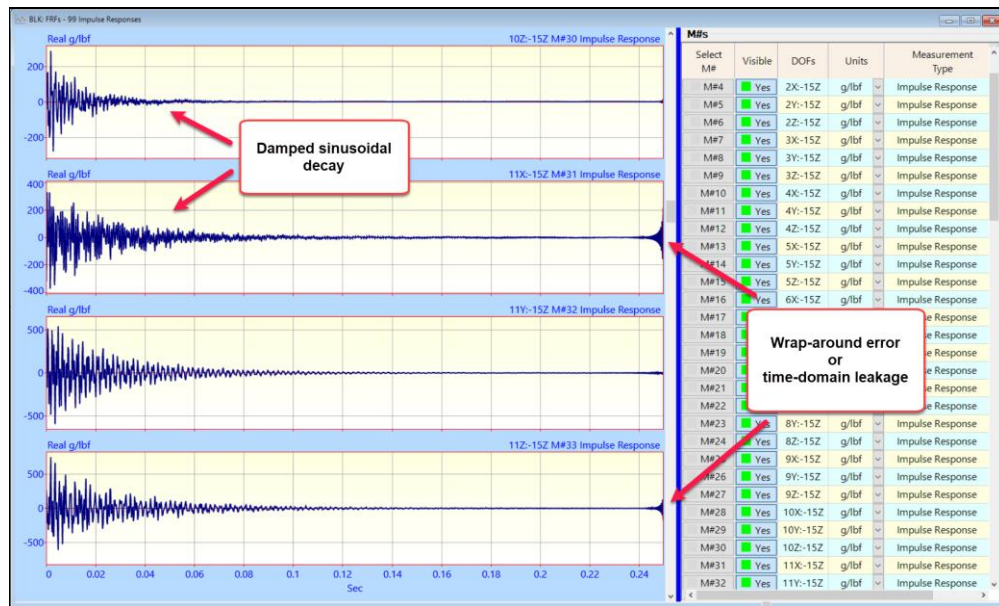
A resonance peak should appear at the same frequency in every **FRF**, except where node points (zero residues) occur.

- Execute **Format | Overlay** and look for a *resonance peak* at the *same frequency* in all or most **FRFs**.

2. Inspect the Impulse Response Functions (IRFs)

Execute **Transform | Inverse FFT** to transform *each FRF* into its *corresponding IRF*.

- Each IRF should exhibit a *damped sinusoidal decay* to *almost zero* at the end of each time waveform, as shown below



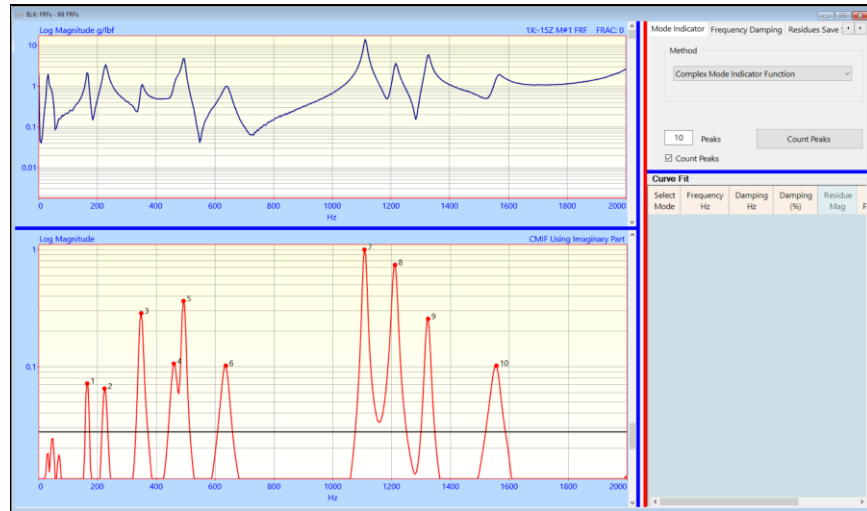
Impulse Response Functions.

- **Wrap around error** (shown above) is not harmful to frequency domain curve fitting

3. Use the Mode Indicator to Count Resonance Peaks

- **Press the Count Peaks** button on the **Mode Indicator** tab to count the number of resonance peaks (modes) in a cursor band or execute **Curve Fit | Mode Indicator | Count peaks**

The peaks counted are numbered on the Mode Indicator as shown below.



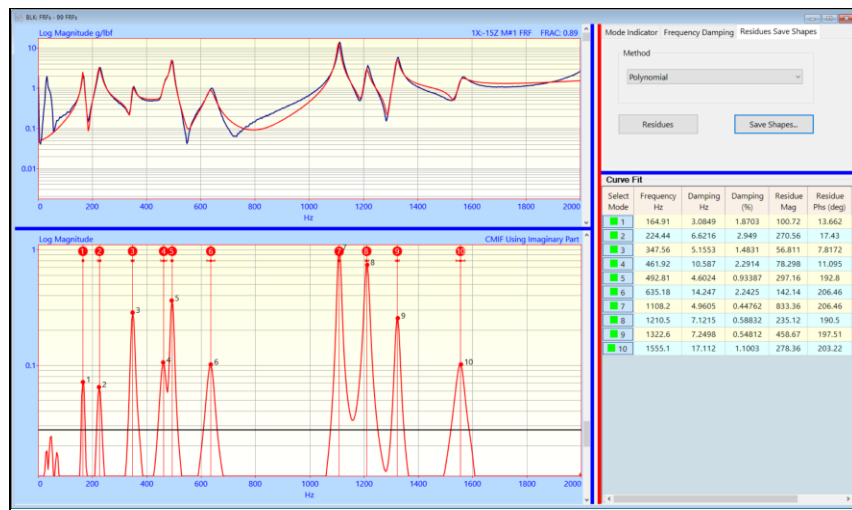
Mode Indicator Showing 10 Counted Peaks Above the Noise Threshold.

4. Use Quick Fit First

- Execute **Curve Fit | Quick Fit** and inspect the results.

Modal parameters for each resonance peak are listed on the **Modal Parameters spreadsheet on the right**.

The **red Fit Function** is also **overlaid** on each **FRF**, as shown below.



5. Use the Band Cursor & Quick Fit

Curve fit only those portions of the data that contain valid resonance peaks.

- If the **Band cursor** is displayed, only data in the cursor band is used for curve fitting
- Enclose a group of resonance peaks with the **Band cursor** and execute **Curve Fit | Quick Fit** for each Cursor band

6. Verify Fundamental Mode Shapes with the Animated Display

The lowest frequency **flexible-body modes** have simple bending and torsional mode shapes.
 Points that deflect substantially different from neighboring Points are indications of poor measurements, poor curve fits, or both.

1. Estimate modal parameters for a few of the **lower frequency (fundamental)** modes
2. Save the results into a Shape Table (**SHP**) and display the mode shapes in animation to verify their mode shapes.

7. Compare Results from Different Curve Fitting Methods

- Curve fit the **FRFs** using more than one curve fitting method

To compare mode shapes from different methods.

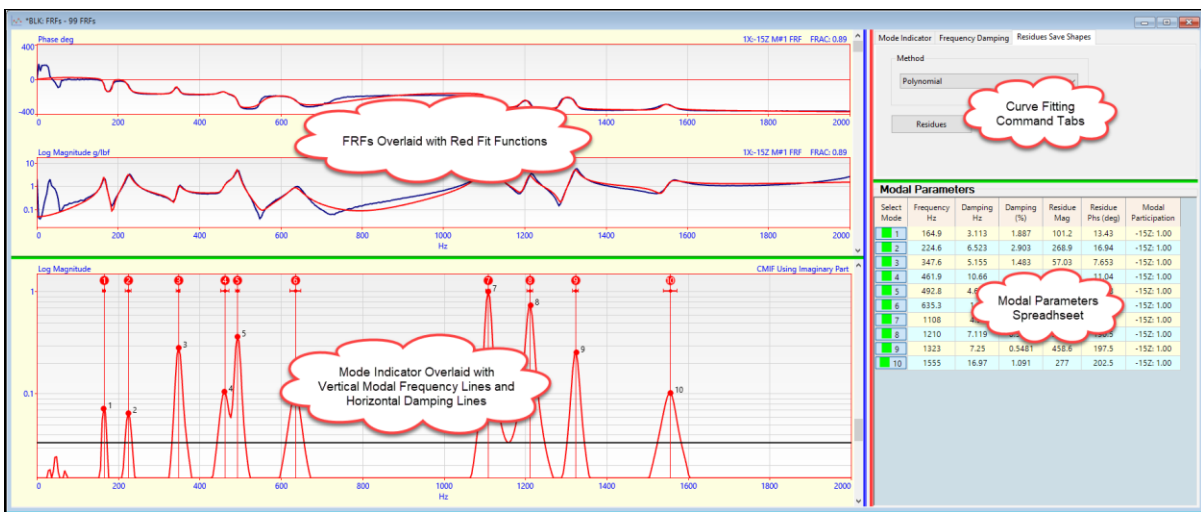
- Execute **Display | MAC** (Modal Assurance Criterion) to numerically compare pairs of mode shapes
- Execute **Draw | Animate a Pair** to display shapes from two different curve fitting methods

Curve Fit | Open Curve Fitting

Enables & disables **FRF**-based curve fitting in a frequency domain Data Block (**BLK**).

When **checked**, curve fitting is **enabled**, and the following changes take place in the Data Block (**BLK**) window

- Data Block (**BLK**) **M#s** are displayed on the upper left side of the window
- A Mode Indicator graph is displayed on the lower left side of the window
- Mode Indicator, Frequency Damping, and Residues Save Shapes tabs are displayed on the upper right side of the window, separated from the graphics by a vertical red splitter bar
- A Modal Parameters spreadsheet is displayed lower right side of the window, separated from the tabs by a horizontal blue splitter bar
- Curve Fitting commands are enabled in the **Curve Fit** menu



Data Block (BLK) Window during Curve Fitting.

During curve fitting, **four splitter bars** are displayed in the Data Block (**BLK**) window

Vertical Splitter Bars

The **vertical blue splitter bar** separates the **M#** graphics & Mode Indicator *on the left* from the **M#s** spreadsheet *on the right*.

- Drag the **vertical blue splitter bar** horizontally to display the **M#s** spreadsheet

The **vertical red splitter bar** separates the **M#** graphics and Mode Indicator *on the left* from the Curve Fitting tabs and Modal Parameters spreadsheet *on the right* of the window.

- **Drag** the **vertical red splitter bar** horizontally to change the size of the Curve Fitting tabs or the **M#** graphics area

Horizontal Splitter Bars

The **left green splitter bar** separates the **M#s** graph and the Mode Indicator graph.

- **Drag** the **left green splitter bar** vertically to change the size of the **M#** graphics or the **Mode Indicator**

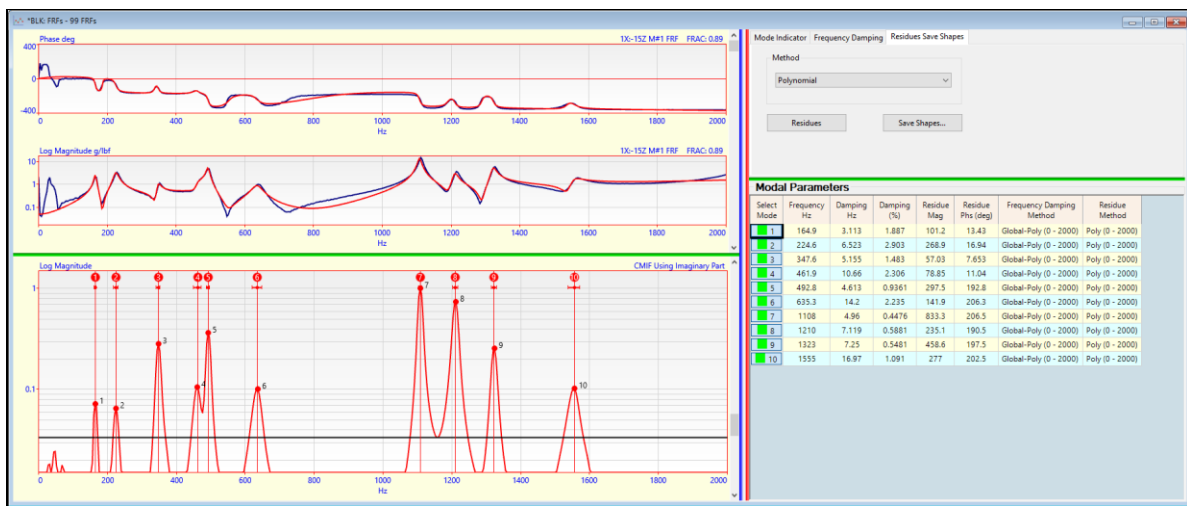
The **right green splitter bar** separates the Curve Fitting tabs and the Modal Parameters spreadsheet.

- **Drag** the **left green splitter bar** vertically to change the size of the Curve Fitting tabs or the Modal Parameters spreadsheet

Modal Parameters Spreadsheet

A list of *all* modal parameter estimates.

- Each *row* of this spreadsheet contains modal parameter estimates *for one mode*



Modal Parameters Spreadsheet

Select Mode Column

- **Click** on the **Select Mode** button to *toggle* the mode selection
- **Double click** on the **Select Mode** column heading to *toggle the selection* of all modes

Frequency & Damping Columns

The *damped natural frequency* is listed in the Frequency (Hz) column.
 Modal damping is listed in two columns, either as the *half-power point damping* in the **Damping (Hz)** column or as the *percent of critical damping* in the **Damping %** column.

Residue Magnitude & Phase Columns

The **Residue** for each mode is listed as *magnitude & phase*

- The magnitude units are the **FRF** units x (radians per second)
 - If the **FRF** units are (g/ N), the **Residue** units are (g/ N-sec)
- Phase units are in *degrees*

Methods Columns

The **Frequency Damping Method** column lists the curve fitting methods used to *estimate frequency & damping* of each mode and the frequency band of data used

The **Residue Method** column lists the curve fitting method used to *estimate the Residue* of each mode and the frequency band of data used

Showing & Hiding Spreadsheet Columns

All spreadsheet columns can be shown or hidden, except the **Select Mode** column.

- **Right click** on any spreadsheet and select **Show Hide Columns** from the menu

The **File | Data Block Options** box will open displaying the **Show Hide** tab.

- **Check** columns to show them and **un-check** columns to hide them

Default Spreadsheet Column Widths

- **Right click** on any spreadsheet and execute **Reset Column Widths** to set its default column width

Spreadsheet Text Cells

- **Select** the text in one or more spreadsheet cells
- **Hold down the Ctrl key** and
 - **Press the X key** to **cut** the *selected* text to the Clipboard
 - **Press the C key** to **copy** the *selected* text to the Clipboard
 - **Press the V key** to **paste** text from the Clipboard into the *selected* cells

Mode Indicator Tab

The first step of modal parameter estimation is to determine *how many modes* are represented by resonance peaks in a set of measurements.

The **Mode Indicator** tab is used to,

1. **Calculate** a Mode Indicator function
2. **Count** the peaks above a **Noise Threshold Line** on the Mode Indicator, as shown below

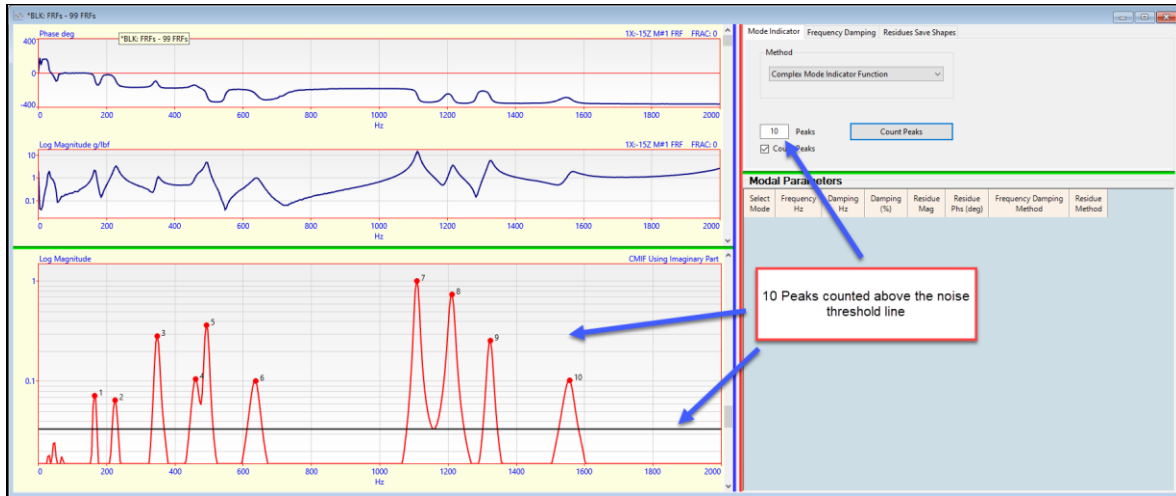
The **Peaks box** on the **Mode Indicator** tab contains the number of peaks counted

To calculate the **Mode Indicator** function and count its peaks,

1. Choose an Indicator from the **Method** list on the **Mode Indicator** tab
2. **Press** the **Count Peaks** button or execute **Curve Fit | Mode Indicator | Count Peaks**

A dialog box will open for choosing a part of the **M#** data to use for calculating the Mode Indicator.

- Choose the **Imaginary** part if the **FRFs** have response units of **Displacement** or **Acceleration**
- Choose the **Real** part if the **FRFs** have response units of **Velocity**.
- Choose **Magnitude** for all other types of **M#s**.



Mode Indicator Tab

After the Mode Indicator is calculated, its peaks are counted *above the horizontal noise threshold line*.

Each modal peak is indicated on the Mode Indicator graph with a **red dot** and a *number next to it*

The number of peaks counted is displayed in the **Peaks** box

Resonance peaks are re-counted on the Mode Indicator when,

- The Band cursor is *moved*
- The noise threshold is *scrolled*
- A new Mode Indicator is *calculated*
- **Curve Fit | Mode Indicator | Smooth Indicator** is executed to *smooth* the Mode Indicator

Frequency & Damping Curve Fitting Methods

Polynomial Method

The **Polynomial** method is **Multi-Degree-Of-Freedom (MDOF)** method that *simultaneously estimates* the modal parameters of one or more modes.

A *least squared error* curve fit is performed on the **FRF** data to obtain estimates of the coefficients of the **FRF** denominator polynomial, called the **characteristic polynomial**.

Modal frequency & damping estimates are then extracted as the *roots* of the **characteristic polynomial**.

Global Curve Fitting

Each resonance peak in an **FRF** is evidence of *at least one mode*.

All **FRF** measurements taken from the same structure should have a *resonance peak* at the *same frequency* for each resonance.

If *multiple FRFs* are *overlaid*, each resonance peak should appear at the *same frequency* in all **FRFs**.

Non-Stationary Data

If multiple **FRFs** are acquired under *non-stationary conditions* (different mass loading due to roving accelerometers, temperature changes, etc.), when the **FRFs** are overlaid, the resonance peaks for a mode *may not line up at the same frequency*.

Global Versus Local Curve Fitting

Global curve fitting can be used when resonance peaks *line up at the same frequency* in a set of overlaid FRFs.

Local curve fitting should be used when resonance peaks *do not line up at the same frequency* in a set of overlaid FRFs.

Frequency Damping Tab

Contains curve fitting methods for estimating the modal frequency & damping of each resonance peak in the FRFs.

If **Count Peaks** was *checked*, the number of modes in the **Modes** box is *the same as* the number of peaks counted in the **Peaks** box on the Mode Indicator tab.

When the **Frequency Damping** button is *pressed* or **Curve Fit | Frequency Damping | Frequency Damping** is executed, the *current Method* chosen on the tab is used to calculate estimates for the number of modes in the **Modes** box

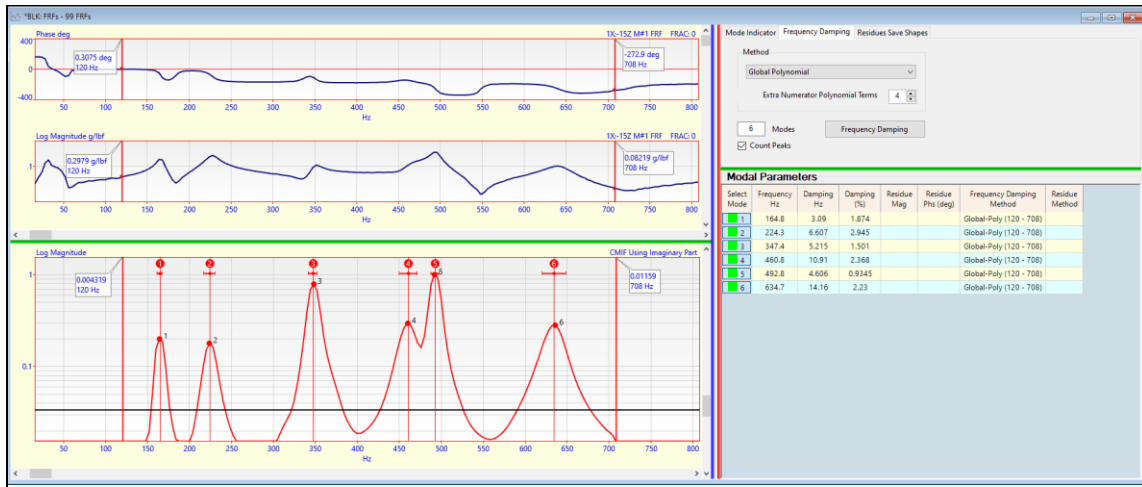
All modal frequency & damping estimates *are added to* the **Modal Parameters** spreadsheet

Global Polynomial Method

Estimates *a global frequency & damping value* for *each mode* by curve fitting *all FRFs*.

Local Polynomial Method

Estimates *a local frequency & damping value* from *each FRF* for *each mode*.



Frequency & Damping Estimates for Six Modes in a Cursor Band.

Vertical Frequency Lines

Each frequency estimate in the **Modal Parameters** spreadsheet is displayed as a **vertical blue line** on the Mode Indicator

- If a mode is *selected*, a **vertical red line** is displayed at each modal frequency on the Mode Indicator
- In the **Band cursor** is displayed, the modes in the band are *selected*

Horizontal Damping Lines

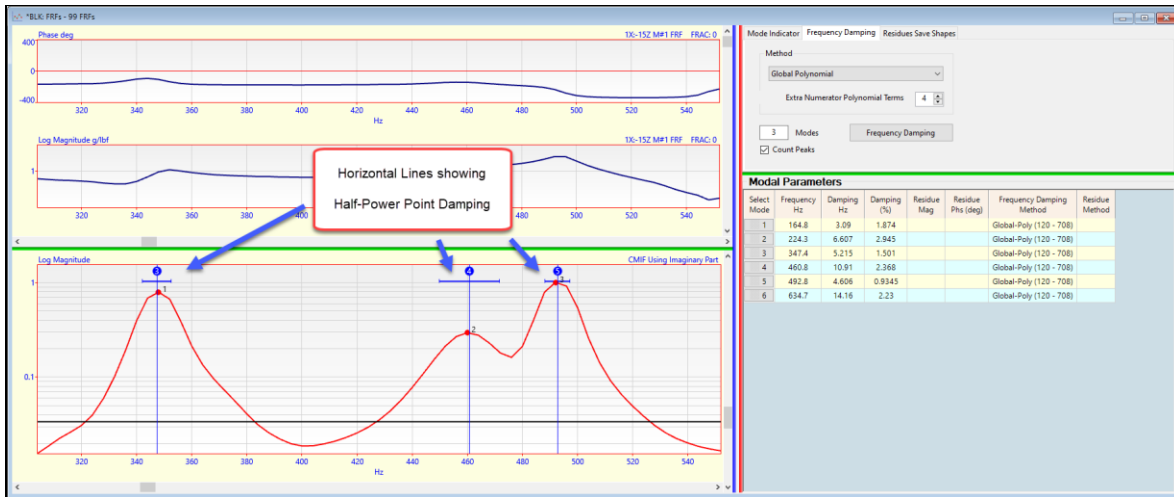
Each modal damping estimate is displayed as a **horizontal blue line** crossing at the **top** of the vertical frequency line.

The length of the **horizontal blue line** is **twice** the modal damping (2σ).

σ is called the **half-power point damping** (Hz), the **3 dB point damping**, or the **damping decay constant**.

2σ is **approximately** equal to the **width of the resonance peak at 70.7 % of the FRF peak magnitude**, or **half of the peak magnitude squared**

- Spin the mouse wheel to Zoom-In and display the horizontal damping lines more clearly



Zoomed Display Showing Half-PowerPoint Damping Lines.

Extra Numerator Polynomial Terms

The residual effects of **out-of-band modes** are compensated for by using **additional numerator polynomial terms** in the Polynomial curve fitting methods.

The number of **extra numerator polynomial terms** is user-specified on the Frequency Damping tab

- Extra polynomial terms are **between 0 & 10**

In most cases, **Four (4) extra terms** is enough terms

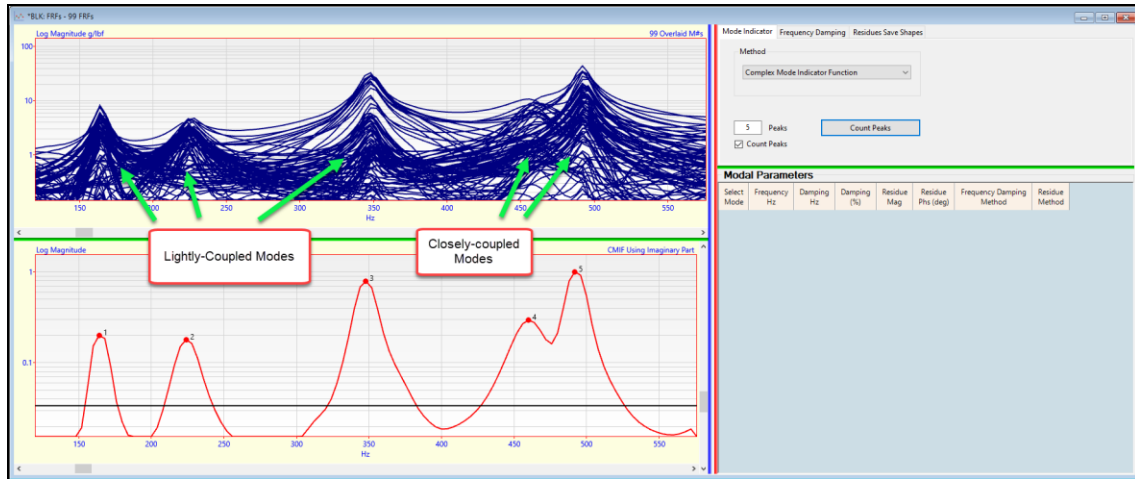
Residue Curve Fitting Methods

Lightly-Coupled Modes

Modes are called **lightly-coupled** if their resonance peaks are **widely separated** in a set of **FRFs**.

When modes are **lightly-coupled**, a **Single-Degree-Of-Freedom (SDOF)** method can be used to estimate modal parameters without incurring significant errors

SDOF methods are used to quickly obtain estimates with a minimum of user interaction



Lightly-Coupled and Closely-Coupled Modes

Closely-Coupled Modes

Modes are called *closely-coupled* if *two or more* modes are represented by *closely-spaced resonance peaks* in a set of **FRFs**.

A Multi-Degree-Of-Freedom (**MDOF**) method should be used to *simultaneously estimate* the modal parameters of *closely-coupled modes*.

Peak Method

The **Peak** method is an **SDOF** method that saves the **peak value** in a band surrounding each modal frequency as the **Residue** for that mode.

The **Peak** method is useful when resonance peaks *shift slightly* from one **FRF** to another, due to *non-stationary* effects

- If the **Imaginary** part of the **FRF** is *displayed*, the *peak value* of the **Imaginary** part is used as the *Residue estimate* for each mode
- If the **Real** part of the **FRF** is *displayed*, the *peak value* of the **Real** part is used as the *Residue estimate* for each mode
- Otherwise, the **Real & Imaginary** parts *at the peak* are used as the *Residue estimate* for each mode

Polynomial Method

When the polynomial method is used for **Residue** curve fitting, the *numerator polynomial* is estimated by a *least squared error* curve fitting process *on each FRF*.

- Following the curve fitting process, the *Residue estimate for each mode* is obtained by performing a *partial fraction expansion* of each *numerator polynomial*

Extra Numerator Polynomial Terms

When the polynomial method is used for **Residue** curve fitting, the residual effects of *out-of-band modes* are compensated for by using *additional numerator polynomial* terms.

The *number of extra polynomial terms* is specified on the **Frequency Damping** tab.

Residues Save Shapes Tab

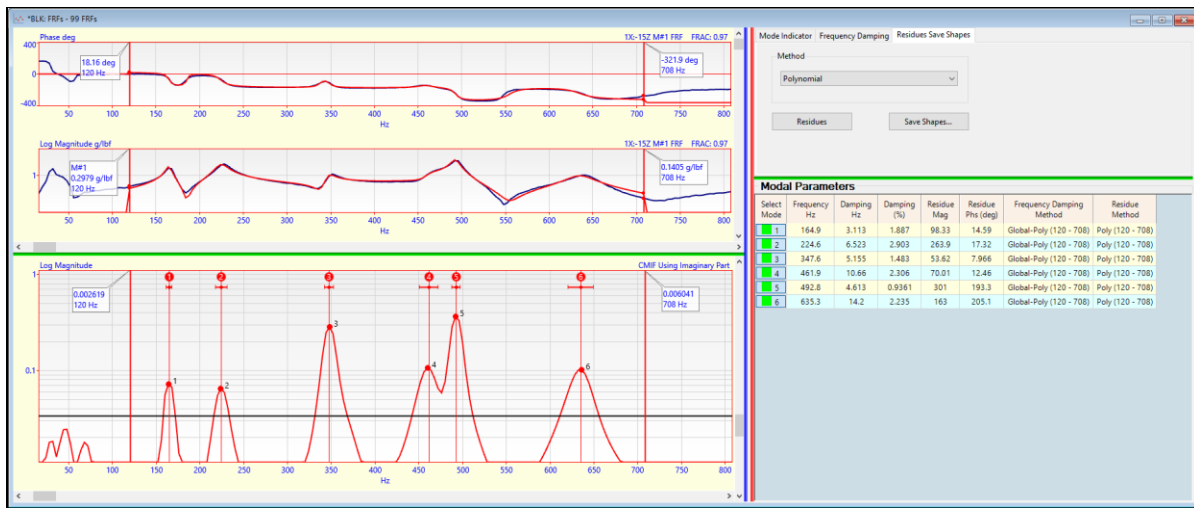
When the **Residues** button is *pressed*, the curve fitting method chosen from the **Methods** list is used to estimate a complex **Residue** (magnitude & phase) for *each mode* and *each FRF*.

- All (or *selected*) **FRFs** are curve fit for *all* (or *selected*) modes in the **Modal Parameters** spreadsheet
- The Residue estimates for *all* (or *selected*) modes are added to the **Modal Parameters** spreadsheet

Fit Function

After residues are estimated, a **red fit function** is *synthesized* using all the modal parameters.

- Each **red fit function** is *overlaid* on each **M#** in the *upper left graphics area*
- Each **red fit function** *should closely match* its corresponding **FRF** over the curve fitting band



Curve Fitting After Residues Have Been Estimated.

- Use the **vertical scroll bar** next to the **M#** graphics to display each **FRF**, its **red fit function**, and its modal parameters in the **Modal Parameters** spreadsheet.
- Use several different display formats (**Magnitude, Bode, Nyquist**) to compare the **FRF** and its **red fit function**.

Save Shapes Button

This button is *enabled* when *at least one mode* has a **Residue** estimate listed in the **Modal Parameters** spreadsheet.

- When the **Save Shapes** button is *pressed*, modal parameters for *all* (or *selected*) modes and *all* (or *selected*) **M#s** are saved into a Shape Table (**SHP**)

Exponential Window Damping Removal

If an exponential window was applied to the **FRFs**, each modal damping estimate will have an *amount of artificial damping* added to it.

When modal parameters are saved into a Shape Table (**SHP**), the amount of artificial damping is subtracted from all modal damping estimates.

The amount of artificial damping due to exponential windowing is displayed in the Window Value column of the **M#s** spreadsheet.

Residue Mode Shapes

When the **Save Shapes** button is *pressed*, **Residue** mode shapes are saved into the Shape Table (**SHP**).

A set of **Residue** mode shapes estimated from *calibrated FRFs* is called a **Modal Model**.

A **Modal Model** preserves the dynamic properties (*mass, stiffness, & damping*) of the structure and can be used for MIMO and SDM analyses.

Curve Fitting OMA Measurements

Operational Modal Analysis (**OMA**) is the process of extracting modal parameters (called **operational mode shapes**) from a set of output-only measurements.

Output-only Measurements

Fourier spectra, Cross spectra & ODS-FRFs can be calculated from output only, response only, or operating data where the excitation forces are not measured.

Flat Force Spectrum

FRF-based curve fitting is applied mainly *around the resonance peaks* in a Fourier spectrum, Cross spectrum, or **ODS-FRF**.

If the frequency spectrum of the *un-measured excitation forces* is assumed to be *relatively flat around each resonance peak*, then **operational mode shapes** can be extracted from *output-only* measurements using **FRF-based curve fitting** methods.

Curve Fitting Fourier Spectra

A Fourier spectrum is the FFT of *single channel* time waveform

If all response time waveforms are *simultaneously acquired*, then **operational mode shapes** can be extracted from a set of Fourier spectra using **FRF-based curve fitting** methods.

Fourier spectra obtained from an **ODS Video** can be curve fit to obtain operational mode shapes.

Curve Fitting Cross Spectra

A Cross spectrum is a *cross-channel measurement* that is calculated between *two channels* of response data.

After a **DeConvolution** window has been applied to a set of Cross spectra, **operational mode shapes** can be extracted from them using **FRF-based curve fitting** methods

Multiple Measurement Sets of Cross spectra can be calculated from one or more *simultaneously acquired* Roving responses and the *same fixed Reference* response

Curve Fitting ODS-FRFs

An **ODS-FRF** is a *hybrid cross-channel* measurement, the magnitude of which is the Auto spectrum of a Roving response and the phase of which is from the Cross spectrum between the Roving and a *fixed Reference* response.

The *correct relative phase* between all Roving responses is preserved if *all ODS-FRFs are calculated* using the *same fixed Reference* response

After a **DeConvolution** window has been applied to a set of **ODS-FRFs**, **operational mode shapes** can be extracted from them using **FRF-based curve fitting** methods

Multiple Measurement Sets of **ODS-FRFs** can be calculated from one or more *simultaneously acquired* Roving responses and the *same fixed Reference* response

Data Block (BLK) Curve Fit Menu

Curve Fit | Quick Fit

Curve fits *all* (or *selected*) **M#s** using the *currently selected methods* on the Mode Indicator, Frequency Damping, and Residues Save Shapes tabs.

- If the **Band cursor** is displayed, *only the data in the band* is used for curve fitting

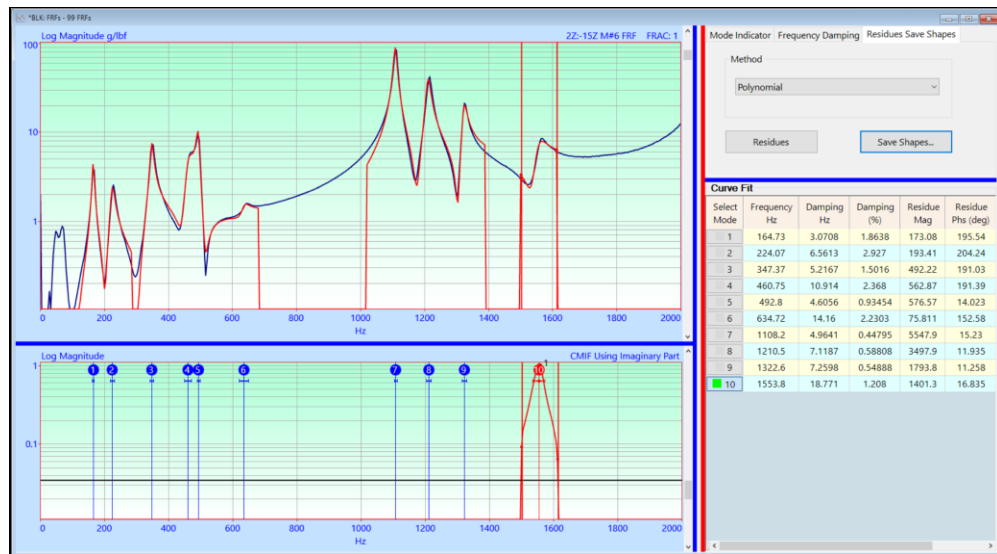
Quick Fit Steps

1. If no Mode Indicator is displayed, a Mode Indicator is calculated using the *current Method* on the **Mode Indicator** tab
2. Peaks above the **noise threshold line** on the Mode Indicator are counted
3. Modal Frequency & Damping are estimated for the number of peaks counted, using the *current Method* on the **Frequency Damp** tab
4. Modal Residues are estimated for the modes estimated in the previous step, using the *current Method* on the **Residues Save Shapes** tab
5. A **red Fit Function** is synthesized and overlaid on the experimental **FRF** data

Improving Quick Fit Results

If some **Quick Fit** results are not satisfactory,

1. Execute **Curve Fit | Modal Parameters | Delete Modes** to remove the *selected* modes from the Modal Parameters spreadsheet
2. Display the Band cursor or change its position to *enclose fewer resonance peaks*
3. Execute **Curve Fit | Mode Indicator | Smooth Indicator** to remove noise peaks from the Mode Indicator
4. Scroll the noise threshold line on the Mode Indicator so that *only resonance peaks are counted*
5. Execute **Curve Fit | Quick Fit again**

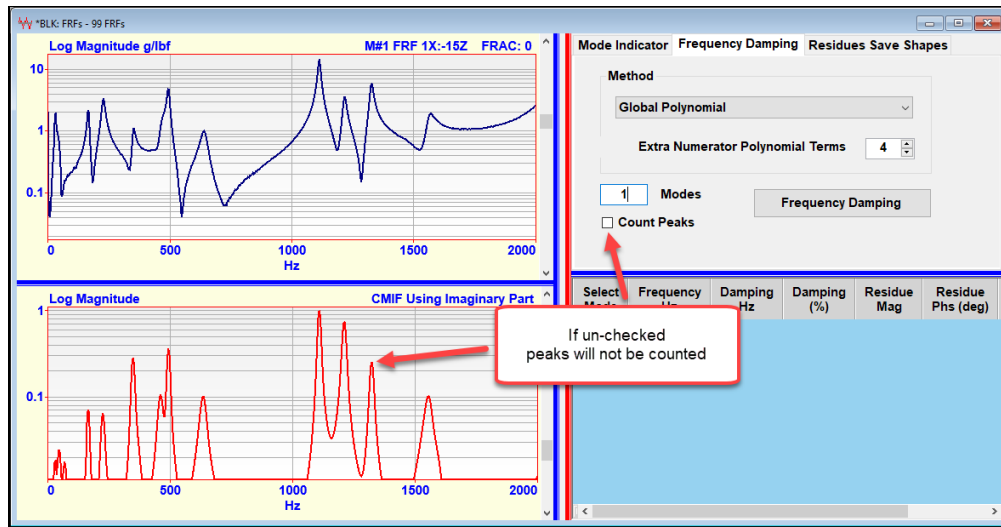


Data Block (BLK) Window Following Quick Fits in Several Bands.

Count Peaks Un-Checked

Quick Fit estimates modal parameters for the number of modes in the **Modes** box on the **Frequency Damping** tab.

- To estimate Frequency & Damping for a **fixed number of modes**
 - **Un-check** the **Count Peaks** box on the **Frequency Damp** tab
 - Enter a number into the **Modes** box
 - Execute **Curve Fit | Quick Fit** or press the **Frequency Damping** button



Curve Fit | Delete All Fit Data

All curve fitting data is saved with the Data Block (**BLK**) in which curve fitting was performed. Before starting a new session of curve fitting, it is a good practice to delete all curve fitting data from the Data Block (**BLK**).

When this command is executed, the following data is deleted,

- All modal parameters in the **Modal Parameters** spreadsheet
- All **Fit Functions**
- All **Mode Indicators**

Curve Fit | Mode Indicator | Count Peaks

Calculates a Mode Indicator, counts its peaks *above the noise threshold line* on a Mode Indicator graph, and sets the number peaks counted in the **Peaks** box.

Executing this command is the *same as pressing the Count Peaks* button on the **Mode Indicator** tab.

Curve Fit | Mode Indicator | Clear Indicator

Clears (zeroes) the Mode Indicator in the *lower left* graphics area.

- The Mode Indicator is used by the **Polynomial** methods on the **Frequency Damping** tab
- If the **Band cursor** is displayed, the Mode Indicator is cleared in the cursor band

Curve Fit / Mode Indicator / Smooth Indicator

Smooths the Mode Indicator to remove noise peaks.

- An *exponential window* is applied to the Mode Indicator to smooth it

See **Transform | Window Data | Exponential** command for details.

- Each time this command is executed, more noise peaks are removed from the Mode Indicator, but the resonance peaks will *become wider*

Curve Fit / Mode Indicator / CMIFs

Calculates and displays the **Complex Mode Indicator Functions (CMIFs)** in Mode Indicator area of the Data Block (**BLK**) window.

Curve Fit / Mode Indicator / MMIFs

Calculates and displays the **Multivariate Mode Indicator Functions (MMIFs)** in Mode Indicator area of the Data Block (**BLK**) window.

Curve Fit / Mode Indicator / Copy CMIFs

Copies the **CMIFs** (including their **Modal Participation** curves) into another Data Block (**BLK**).

Curve Fit / Mode Indicator / Copy MMIFs

Copies the **MMIFs** (including their **Modal Participation** curves) into another Data Block (**BLK**).

Curve Fit / Frequency Damping / Number of Modes

Sets the *number of modes* to be used by the Frequency Damping curve fitting methods.

Curve Fit / Frequency Damping / Frequency Damping

Estimates modal *frequency & damping* for the current number in the **Modes** box on the **Frequency Damping** tab, using *all* (or *current Band cursor*) data from *all* (or *selected*) **M#s** in the Data Block (**BLK**).

Curve Fit / Residues

Estimates modal *residues* using *all* (or *selected*) modes in the **Modal Parameters** spreadsheet, using data from *all* (or *selected*) **M#s** in the Data Block (**BLK**).

Executing this command is the *same as pressing the Residues button* on the **Residues Save Shapes** tab.

Curve Fit / Modal Parameters / Sort by Frequency

Sorts the modes in the **Modal Parameters** spreadsheet by *ascending* order of frequency.

Curve Fit / Modal Parameters / Select All Modes, Select None, Invert Selection

These commands are used for *selecting* modes in the **Modal Parameters** spreadsheet

Curve Fit / Modal Parameters / Delete Modes

Deletes the *selected* modes from the **Modal Parameters** spreadsheet.

Curve Fit / Modal Parameters / Save Shapes

Saves *all* (or *selected*) modes from the **Modal Parameters** into a Shape Table (**SHP**).

Executing this command is the *same as pressing the Save Shapes button* on the **Residues Save Shapes** tab.

Curve Fit / Modal Parameters / MAC

Opens the Modal Assurance Criterion (**MAC**) window for comparing shapes in the **Modal Parameters spreadsheet** with shapes in a chosen Shape Table (**SHP**).

Curve Fit / Fit Functions / Clear Fit Functions

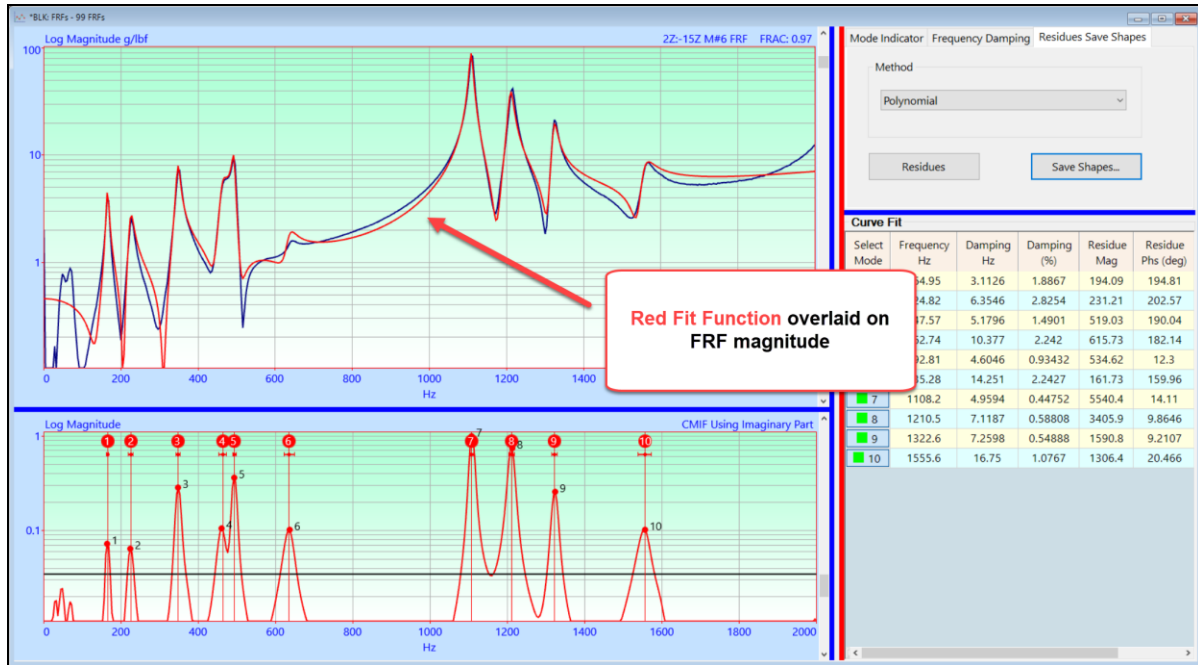
Clears (zeros) the **red Fit Functions** of *all (or selected) M#s*.

- If the Band cursor is displayed, the Fit Functions are *cleared in the band*

Curve Fit / Fit Functions / Synthesize Fit Functions

Synthesizes a **red Fit Function** for *all (or selected) M#s*, using the modal parameters of *all (or selected) modes* in the **Modal Parameters** spreadsheet.

- A **red Fit Function** is *overlaid* on each **M#**
- If the **Band** cursor is displayed, **Fit Functions** are *synthesized in the band*



Synthesized Fit Function Overlaid on an FRF.

Curve Fit / Fit Functions / Fit Functions

Toggles the display of the Fit Functions overlaid on the **FRFs** in the Data Block (**BLK**) window.

Curve Fit / Fit Functions / Copy Fit Functions

Copies the Fit Functions into another Data Block (**BLK**).

Curve Fit / Close Curve Fitting

Terminates curve fitting and removes the **Curve Fitting** tabs from the Data Block (**BLK**) window

Shape Table (SHP) Display Menu

Display / MAC

Calculates and displays **Modal Assurance Criterion (MAC)** values between *pairs of shapes* in two Shape (SHP) Tables.

Each Shape Table (SHP) can contain **ODS's**, **EMA Mode Shapes**, **FEA Mode Shapes**, or Engineering Data (**EDS**) Shapes.

What is MAC?

MAC is a quantitative method for measuring the *co-linearity of two shape vectors, {u} & {v}*.

MAC is calculated between two shapes {u} & {v} with the formula,

$$MAC = \frac{\| \{u\}^h \{v\} \|^2}{\{u\}^h \{u\} \{v\}^h \{v\}}$$

{u} → complex shape (**m-vector**)

{v} → complex shape (**m-vector**)

m → the number of matching DOFs

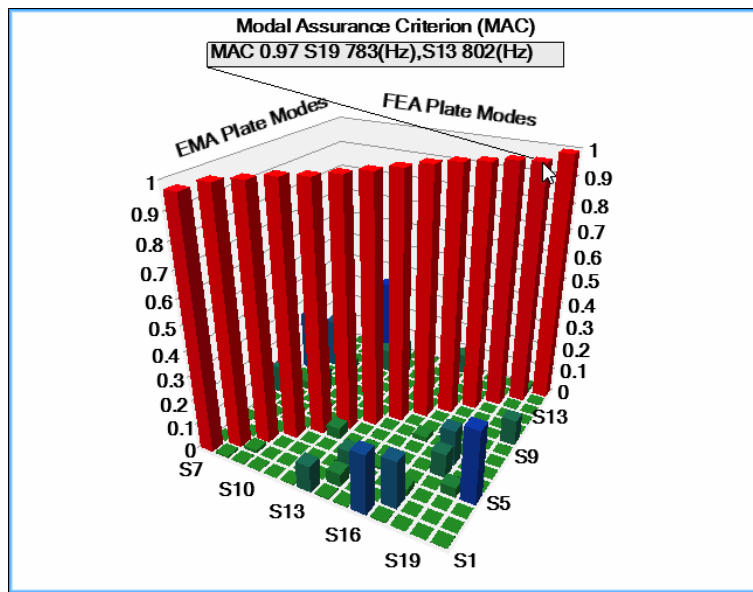
h → transposed conjugate vector

$\| \|^2$ → *magnitude squared* of the vector product

MAC values range *between 0 & 1*.

- **MAC = 1.00** → two shapes are *co-linear*, they both lie on the same straight line
- **MAC > 0.90** → two shapes are *similar*
- **MAC < 0.90** → two shapes are *different*

Only *shape components with matching DOFs* in the two Shape (SHP) Tables are used to calculate **MAC** values.



MAC 3D Bar Chart.

What is CoMAC?

Coordinate Modal Assurance Criterion (**CoMAC**) is a quantitative method for comparing shape components (shape **DOFs**) *between pairs of shapes* in two Shape (**SHP**) Tables.

Each Shape Table (**SHP**) can contain **ODS**'s, **FEA** Mode Shapes, **EMA** Mode Shapes, or Engineering Data (**EDS**) Shapes.

CoMAC uses the same formula as **MAC**, but $\{u\}$ & $\{v\}$ are vectors of data from *two M#s with matching DOFs*

CoMAC values range *between 0 & 1*

- **CoMAC** = 1.00 → components of *all* (or *selected*) shapes are *co-linear* for a *pair of matching DOFs*
- **CoMAC** > 0.90 → components of *all* (or *selected*) shapes are *similar* for a *pair of matching DOFs*
- **CoMAC** < 0.90 → components of *all* (or *selected*) shapes are *different* for a *pair of matching DOFs*

MAC Window Commands**File | Copy Graphics to Clipboard**

Copies the **MAC** window graphics to the Windows Clipboard.

File | Print

Prints the graphics on the system graphics printer.

File | Close

Closes the **MAC** window.

Display | Spreadsheet

Displays the **MAC** or **CoMAC** values in a spreadsheet.

Display | 3D Bar Chart

Displays the **MAC** or **CoMAC** values in a 3D bar chart.

- *Right click & drag* to rotate the 3D Bar Chart

Display | Values

When *checked*, the **MAC** or **CoMAC** value for one shape pair is displayed on the 3D Bar Chart.

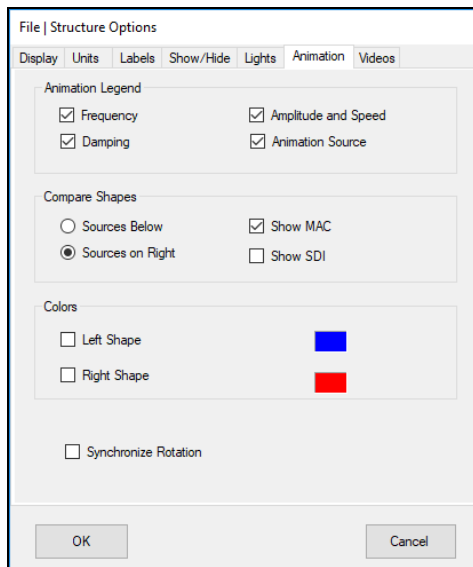
- *Hover* the mouse pointer over a bar to display its value

Display | MAC, CoMAC

When *checked*, either **MAC** or **CoMAC** values are displayed in either a 3D Bar Chart or a spreadsheet.

Structure Options Animation Tab

If **Show MAC** is *checked* on the **Animation** tab in the **File | Structure Options** box, the **MAC** value between two shapes is displayed during *animation of a pair* of shapes.



Display / SDI

Calculates and displays **Shape Difference Indicator (SDI)** values between *pairs of shapes* in two Shape (SHP) Tables.

Each Shape Table (SHP) can contain **ODS's**, **EMA Mode Shapes**, **FEA Mode Shapes**, or **Engineering Data (EDS) Shapes**.

What is SDI ?

SDI is a quantitative measure of the *difference between two shape vectors {u} & {v}*.

SDI is calculated between two shapes {u} & {v} using the formula,

$$SDI = \left(1 - \frac{\| \{u\} - \{v\} \|^2}{\{u\}^h \{u\} + \{v\}^h \{v\}} \right)^2$$

or

$$SDI = \left(\frac{2 \operatorname{real}(\{u\}^h \{v\})}{\{u\}^h \{u\} + \{v\}^h \{v\}} \right)^2$$

real({u}^h{v}) → **real part** of the vector product

{u} → complex shape (**m-vector**)

{v} → complex shape (**m-vector**)

m → the number of matching **DOFs**

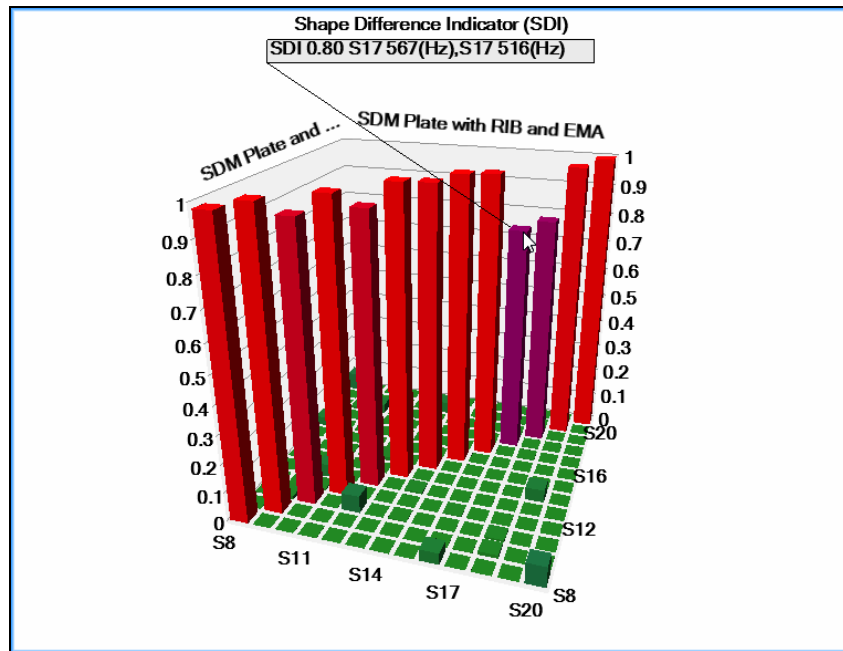
h → transposed conjugate vector

SDI values range between 0 & 1

- SDI = 1.00 → two shapes are *identical*
- SDI > 0.90 → two shapes are *similar*
- SDI < 0.90 → two shapes are *different*

SDI values between *pairs of shapes* in two Shape (SHP) Tables are displayed in a bar chart as shown below.

Only *shape components with matching DOFs* in the two Shape (SHP) Tables are used to calculate SDI.



SDI 3D Bar Chart.

SDI Window Commands

File | Copy Graphics to Clipboard

Copies the SDI window graphics to the Windows Clipboard.

File | Print

Prints the graphics on the system graphics printer.

File | Close

Closes the SDI window.

Display | Spreadsheet

Displays the SDI values in a spreadsheet.

Display | 3D Bar Chart

Displays the SDI values in a 3D bar chart.

- *Right click & drag* to rotate the 3D Bar Chart

Display | Values

When *checked*, the SDI value for one shape pair is displayed on the 3D Bar Chart.

- *Hover* the mouse pointer over a bar to display its value.

Structure Options Animation Tab

If **Show SDI** is *checked* on the **Animation** Tab in the **File | Structure Options** box, the **SDI** value between two shapes is displayed during *animation of a pair* of shapes.

Display | Participation

Displays **Participation** values between *pairs of shapes* in two Shape Table (**SHP**).

What Is Shape Participation?

The **Participation** of vector **{u}** in vector **{v}** is defined by the equation

$$\{u\} (\text{Participation}) = \{v\}$$

Participation is calculated as the *least-squared-error solution* to the above equation with the formula

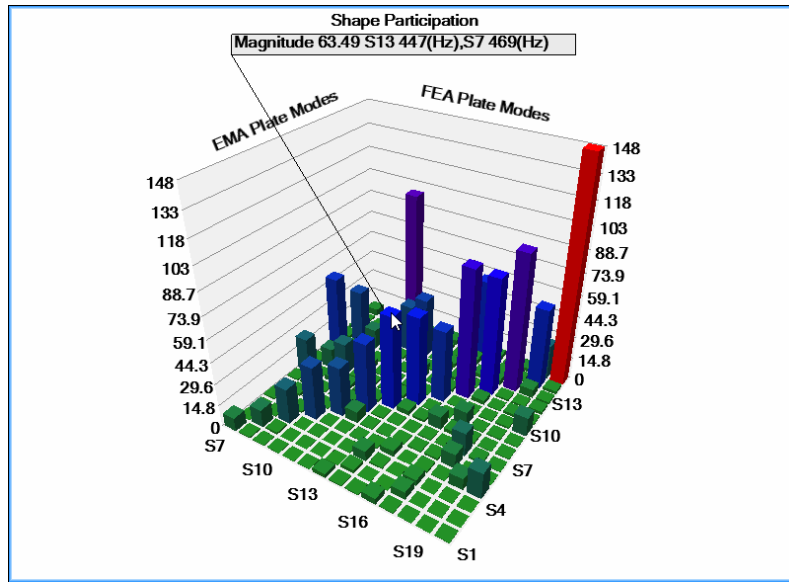
$$\text{Participation} = \frac{\{u\}^h \{v\}}{\{u\}^h \{u\}}$$

{u} → complex shape (**m-vector**)

{v} → complex shape (**m-vector**)

m → the number of matching **DOFs**

h → indicates the transposed conjugate vector



Bar Chart of Participation Magnitudes.

Participation Window Commands

File | Copy Graphics to Clipboard

Copies the **Participation** window graphics to the Windows Clipboard.

File | Print

Prints the window graphics on the system graphics printer.

File | Close

Closes the **Participation** window.

Display | Spreadsheet

Displays the **Participation** values in a spreadsheet.

Display | 3D Bar Chart

Displays the **Participation** values in a 3D bar chart.

- **Right click & drag** to rotate the 3D Bar Chart

Display | Value

When **checked**, the **Participation** value for one shape pair is displayed on the 3D Bar Chart.

- **Hover** the mouse pointer over a bar to display its value

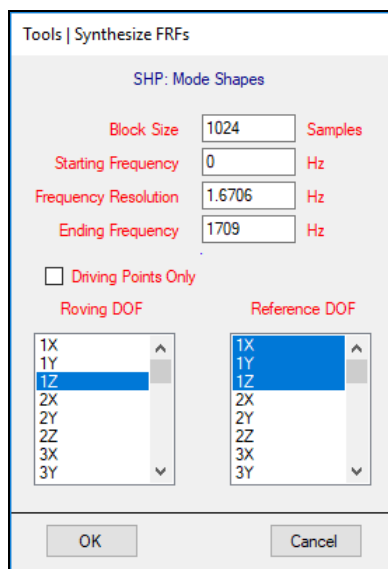
Display | Real Part, Imaginary Part, Magnitude

When **checked**, displays the Real Part, Imaginary Part or Magnitude of the **Participation** values in either a 3D Bar Chart or a spreadsheet.

Shape Table (SHP) Tools Menu**Tools | Synthesize FRFs**

When this command is executed, a dialog box will open allowing you to choose a Shape Table (SHP) with a Modal Model in it.

- If using **Residue** mode shapes, choose **all desired DOFs** for synthesizing **FRFs**
- If using **UMM** mode shapes, choose **all desired Roving & Reference DOFs** for synthesizing **FRFs**

**Residue mode shapes**

Residue mode shapes are obtained from curve fitting a set of **FRFs**

- Each component of a **Residue** mode shape is defined between **a pair of DOFs (Roving DOF : Reference DOF)**

UMM mode shapes

UMM mode shapes can be obtained from an **FEA** model or by re-scaling a set of **Residue** mode shapes

- **FRFs** can be synthesized for **any two M#s** in a set of **UMM** mode shapes

Type of Shapes in a Shape Table (SHP)	FRFs Synthesized
Residue mode shapes	Only for each shape component
UMM mode shapes	Between any pair of shape components

What is a Modal Model?

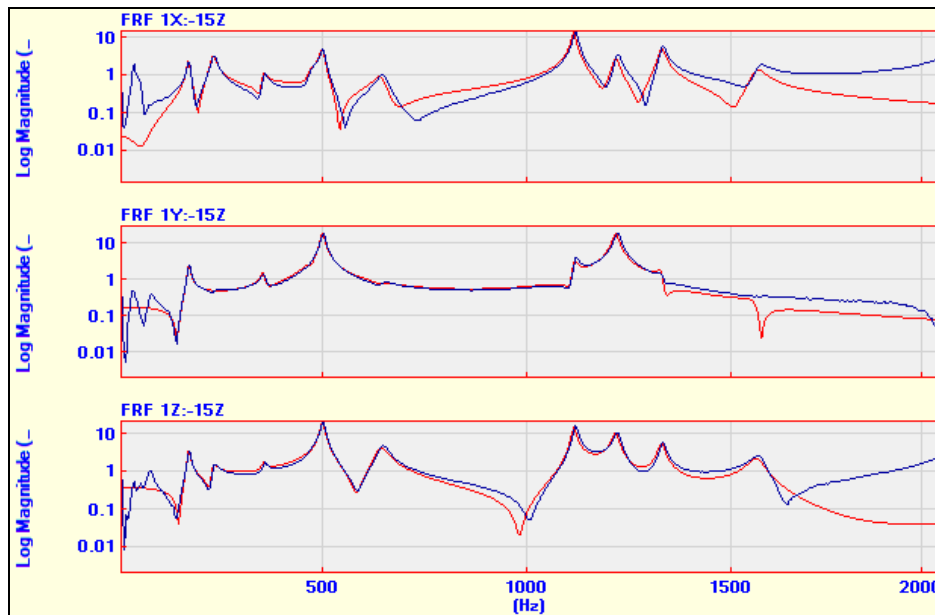
Both **Residue** mode shapes and **UMM** mode shapes are called a **Modal Model** when they are scaled in a manner which preserves the dynamic properties of a structure (its mass, stiffness, & damping properties).

The mode shapes of a Modal Model can be used for

- Synthesizing **FRFs**
- MIMO calculation of multiple Outputs from multiple Inputs, and multiple Inputs from multiple Outputs
- **SDM** including Modal Sensitivity, Sub structuring, Adding Tuned Absorbers
- **FEA** Model Updating

Overlaying Synthesized & Measured FRFs

- **Select** the **measured FRFs** in their Data Block (**BLK**)
- In the Data Block (**BLK**) window containing the **synthesized FRFs**, execute **Edit | Paste M#s from File** to paste the measured **FRFs** together with the synthesized **FRFs**.
- Use the **Color** column in the **M#s** spreadsheet to color the synthesized and measured **FRFs** differently.
- Execute **Format | Overlay by DOF** to overlay the synthesized & measured **FRFs**.



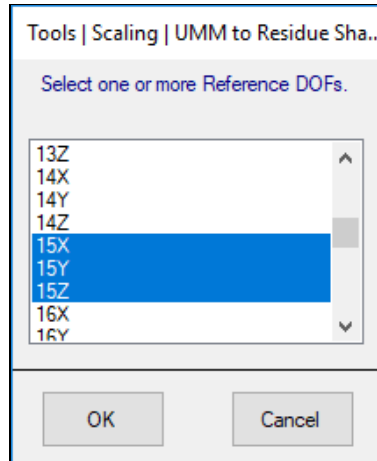
Synthesized & Measured FRFs Overlaid.

Tools / Scaling / UMM to Residue Shapes

Re-scales a set of **UMM** mode shapes into **Residue** mode shapes.

Only *enabled* in a Shape Table (**SHP**) that contains **UMM** mode shapes.

When this command is executed, the following dialog box is opened



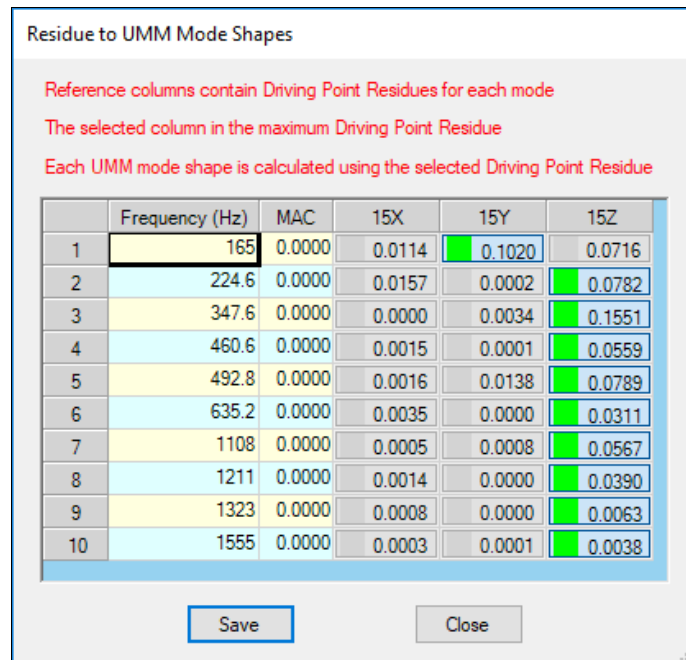
- Select at least one Reference DOF for re-scaling the **UMM** mode shapes into **Residue** mode shapes.

Tools / Scaling / Residues to UMM Shapes

Re-scales a set of **Residue** mode shapes into **UMM** mode shapes.

Only *enabled* in a Shape Table (**SHP**) that contains **Residue** mode shapes.

When this command is executed, in a Shape Table (**SHP**) containing **Residue** mode shapes with *multiple Reference DOFs*, the following dialog box is opened.



- Select at least one Driving Point Residue for each mode to re-scale it

Tools / Scaling / Un-Scaled to Scaled Shapes

Re-scales a set of un-scaled shapes using a set of **Residue** mode shapes or **UMM** mode shapes.

This command is useful for re-scaling **OMA mode shapes** using the following scaled mode shapes,

- **FEA** mode shapes that are scaled as **Residue** mode shapes or **UMM** mode shapes
- **EMA** mode shapes that are scaled as **Residue** mode shapes or **UMM** mode shapes

Tools / Scaling / Rapid Test Residues to UMM Shapes

Re-scales a set of Rapid Test Residues to **UMM** mode shapes.

Residues versus Mode Shape Components

$$\text{Residues} = \begin{Bmatrix} \text{RES}(1:2) \\ \text{RES}(1:3) \\ \text{RES}(2:3) \end{Bmatrix} \quad \text{Mode Shape} = \begin{Bmatrix} \text{MSC}(1) \\ \text{MSC}(2) \\ \text{MSC}(3) \end{Bmatrix}$$

$\text{RES}(1:1) = \text{MSC}(1) \times \text{MSC}(1)$
 $\text{RES}(1:2) = \text{MSC}(1) \times \text{MSC}(2)$
 $\text{RES}(1:3) = \text{MSC}(1) \times \text{MSC}(3)$
 $\text{RES}(2:3) = \text{MSC}(2) \times \text{MSC}(3)$

Single-Reference Modal Test

$$\text{Residue Mode Shape} = \begin{Bmatrix} \text{RES}(1:1) \\ \text{RES}(2:1) \\ \text{RES}(3:1) \end{Bmatrix} = \begin{Bmatrix} \text{MSC}(1) \times \text{MSC}(1) \\ \text{MSC}(2) \times \text{MSC}(1) \\ \text{MSC}(3) \times \text{MSC}(1) \end{Bmatrix}$$

Converting Rapid Test Residues to a Mode Shape

Starting with a Driving Point Residue

$$\text{MSC}(1) = \sqrt{\text{RES}(1:1)} = \sqrt{\text{MSC}(1) \times \text{MSC}(1)}$$

$$\text{MSC}(2) = \text{RES}(1:2) / \text{MSC}(1)$$

$$\text{MSC}(3) = \text{RES}(1:3) / \text{MSC}(1)$$

$$\text{MSC}(4) = \text{RES}(3:4) / \text{MSC}(3) = \text{RES}(4:3) / \text{MSC}(3)$$

Starting with Triangle Point Residues

$$\text{MSC}(1) = \sqrt{\frac{\text{RES}(1:2) \times \text{RES}(1:3)}{\text{RES}(2:3)}}$$

$$= \sqrt{\frac{\text{MSC}(1) \times \text{MSC}(2) \times \text{MSC}(1) \times \text{MSC}(3)}{\text{MSC}(2) \times \text{MSC}(3)}}$$

What is a Rapid Test?

Rapid Test Residues are obtained by curve fitting a set of measurements that were calculated from a **TRN chain** of acquired data.

In a Rapid Test, either the impactor or the response sensor (e.g. the accelerometer) can be moved between acquisitions of data, provided that a **TRN chain** of data is acquired.

A **TRN chain** is formed when each Measurement Set of acquired data contains a **DOF** that is also contained in another Measurement Set.