VES-4600 Advanced Modal Analysis

March 7, 2025

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If the **VES-4600 Advanced Modal Analysis** option is authorized by your MEscope license, the following commands are enabled in MEscope. Execute **Help | License Manager** to verify the Options authorized by your MEscope license.

Additional Structure (STR) Commands

- Animate | Normalize Shapes
- **M#** Links | Interpolate Source **M#s**

Additional Data Block (BLK) Commands

- Display | Complexity Plot
- Display | Magnitude Ranking
- Tools | **Modal Decomposition**
- Tools | **Modal Expansion**
- Curve Fit | **Stability** | **Stability** Clear
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- Curve Fit | **Stability** | **Stability** Reset
- Curve Fit | **Stability** | Poles Selection Box
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Multi-Reference Curve Fitting

- Multi-Reference curve fitting meth**ODS** are added on the Mode Indicator, Frequency Damping, & Residues Save Shapes tabs

Stability diagram

- The **Stability** diagram is enabled on the Mode Indicator display
- **Stability** tabs are added to the Frequency Damping tab
- The **Stability** tab contains several Multi-Reference curve fitting meth**ODS**
- The **Stable Groups** tab contains controls for defining stable groups of poles on the **Stability diagram**

Additional Shape (SHP) Table Commands

- Display | Complexity Plot
- Display | Magnitude Ranking
- Display | **M#s** | Effective Mass, Stiffness, Damping
- Display | Poles
- Tools | Modal Participation
- Tools | Modal Expansion

Additional Shape (SHP) Table Columns

- **MPC** (Modal Phase Co-linearity)
- Modal Participation

Special Mouse & Keyboard Commands

Stability diagram

- ***Click*** near a pole on the **Stability** diagram to display its **Frequency & Damping**
- ***Click*** near the Frequency & Damping of a pole ***to remove it*** from the **Stability** diagram
- ***Click on*** the Frequency & Damping and ***drag to move it*** on the **Stability** diagram
- ***Hold down* the Ctrl Key** and ***click & drag*** to draw a selection box enclosing a group of stable poles on the **Stability** diagram

When Is Multi-Reference Modal Testing Necessary?

Multi-Reference modal testing is required when a structure has resonances that occur under one of the following conditions,

1. **Closely-coupled Modes**: Resonance peaks ***heavily overlapped*** or ***one resonance peak*** for ***two or more modes***.
2. **Repeated Roots**: Two or more modes have the ***same natural frequency*** but ***different mode shapes***.
3. **Local Modes**: Modes have ***non-zero mode shapes*** only in ***local regions*** of a structure.

Multi-Reference Modal Testing

In a **Multi-Reference Modal Test**, multi-reference **FRFs** are calculated using data that is acquired using ***two or more*** **fixed Reference sensors**.

Multi-reference curve fitting is required to extract all modal parameters from a set of multi-reference **FRFs**.

Single-Reference FRFs

Single-reference **FRFs** are the ***minimum requirement*** for extracting modal parameters of a structure using **FRF**-based curve fitting.

- Single-reference **FRFs** are obtained by exciting the structure with a ***single fixed exciter*** and ***simultaneously acquiring*** the force & response from ***one or more response DOFs***
- Single-reference **FRFs** are also obtained by using a ***single fixed response sensor*** and ***simultaneously acquiring*** force & response while exciting the structure at ***one or more excitation DOFs***

Multi-Reference FRFs

Multi-reference **FRFs** ***are required*** for extracting the modal parameters of **closely-coupled modes**, **repeated roots**, or **local modes** of a structure.

- Multi-reference **FRFs** are obtained by exciting the structure with ***multiple fixed exciters*** and ***simultaneously acquiring*** the forces & responses from ***one or more response*** **DOFs**
- Multi-reference **FRFs** are also obtained by exciting the structure at ***multiple DOFs*** and ***simultaneously acquiring*** the force and responses from ***more than one response point & direction***.

Multi-Reference Modal Testing

A **multi-reference modal test** is done using either ***multiple fixed exciters*** with sensors to measure the forces, or ***multiple fixed response*** sensors.

A multi-reference modal test ***is useful*** when a structure has ***high modal density*** (many resonance peaks in close frequency to one another).

Multiple Shaker Test

Large structures will commonly exhibit ***non-linear dynamic behavior,*** so they are typically tested using multiple shakers, driven by ***pure*** or ***burst*** random excitation signals.

Random excitation together with **spectrum averaging** is used to ***"average out"*** the ***non-linear dynamic behavior*** of the structure from the spectra and hence from the **FRFs**.

- The multiple shakers must be driven with ***uncorrelated broad-band signals***
- An **FRF** is calculated between ***each force & each response***.
- ***Multiple & Partial Coherences*** are also calculated.

Multi-Reference Roving Impact Test

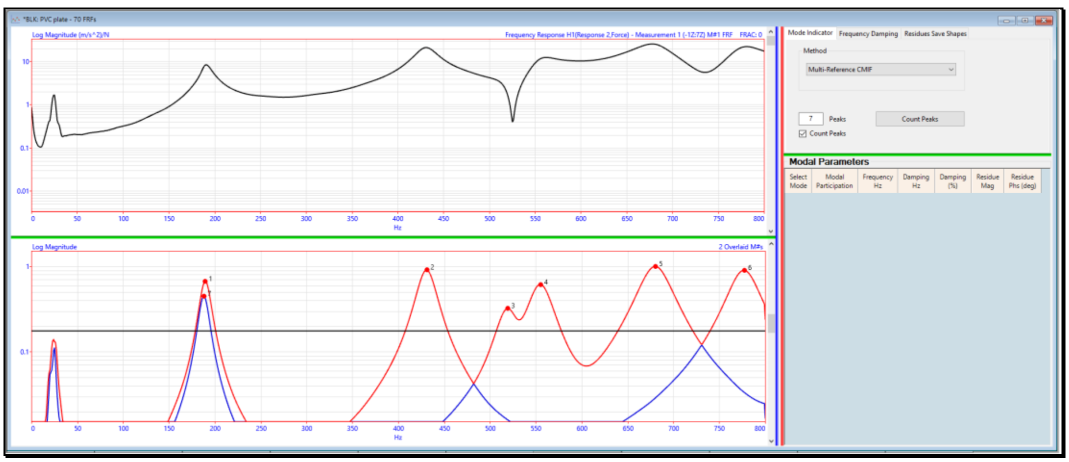
In this test, ***two or more fixed response sensors*** are used, and the structure is excited with a roving impactor.

- This test is the ***same as performing*** **two or more Roving impact tests**
- This test takes ***no more time*** to complete than a single-reference Roving impact test

Multi-Reference Mode Indicator

A Multi-Reference Mode Indicator is used for counting resonance peaks during Multi-reference curve fitting.

- A Multi-Reference Mode Indicator contains a ***separate curve for each reference***
- A peak ***at or near the same frequency in two or more*** **Multi-Reference Indicator curves** indicates ***closely-coupled modes*** or ***repeated roots***



Multi-Ref CMIFs Indicating Two Closely Coupled Modes *Near 200 Hz.*

Multi-Reference CMIF

A **Multi-Reference CMIF** is calculated by performing a ***singular value decomposition*** of multi-reference **FRFs**.

- A ***separate*** **CMIF** curve is calculated ***for each reference***
- ***Each peak*** on a **CMIF** curve ***indicates a resonance***

Multi-Reference MMIF

A **Multi-Reference MMIF** mode indicator is calculated by performing a ***an eigen-solution*** of multi-reference **FRFs**.

- A ***separate*** **MMIF** curve is calculated ***for each reference***
- ***Each peak*** on an **MMIF** curve ***indicates a resonance***

Modal Participation Factor

Each Multi-Reference **CMIF** & **MMIF** also provides a **Modal Participation Factor** curve for each multi-reference **FRF**.

Each Modal participation factor is used to ***weight each reference of* FRF *data*** during Multi-reference curve fitting

Multi-Reference Curve Fitting

MEscope contains several different Multi-Reference Parameter Estimation (curve fitting) meth**ODS**.

- Each curve fitting method uses a ***modal participation curve to weight the FRF data*** during curve fitting

Multi-Reference Polynomial

The Multi-Reference Polynomial method uses a multi-reference version of the **Rational Fraction Orthogonal Polynomial** method.

- The curve fitting model size in the **Modes** box on the **Polynomial** tab is used for estimating modal Frequency & Damping

Stability Diagram Curve Fitting

Why Use a Stability diagram?

A **Stability** diagram is a plot of pole estimates (**modal frequency & damping**) using a ***progression*** of curve fitting model sizes.

A **Stability** diagram ***does not rely on peak counting*** on a Mode Indicator curve.

- **Stability** curve fitting is done for model sizes from **"1"** to the **Max. Model Size** listed on the **Stability** tab
- Each modal **frequency estimate** is displayed as a ***vertical line*** on the **Stability** diagram
- Each modal **damping estimate** is displayed as a ***horizontal line*** on the **Stability** diagram
- The **Stability** tab contains several curve fitting meth**ODS** for estimating modal frequency & damping

AF Polynomial (Alias-Free Polynomial)

This method is an extension of the **Rational Fraction Orthogonal Polynomial** curve fitting method.

It is called ***"alias-free"*** because it places extra computational modes ***toward the edges*** of the ***curve fitting band*** instead of aliasing them throughout the band

Complex Exponential

This popular ***time domain method*** estimates poles by curve fitting **Impulse Response Functions (IRFs)**, the **Inverse FFT** of **FRFs**.

- During curve fitting, the ***Inverse* FFT** is applied to ***each* FRF** to obtain its corresponding IRF

Least Squares Complex Frequency (LSCF)

This method is an extension of the **Rational Fraction Orthogonal Polynomial** curve fitting method and uses the **Z-transform** to obtain more stable powers of the frequency variable.

The **Z-transform** maps frequencies to a ***unit circle***, resulting in ***numerically stable*** solution equations.

Creating a Stability diagram

The **Stability** diagram is displayed on top of a Mode Indicator graph.

- ***Press* the Stability button** on the **Stability** tab or execute **Curve Fit | Stability | Stability diagram**

What is a Stable Pole Group?

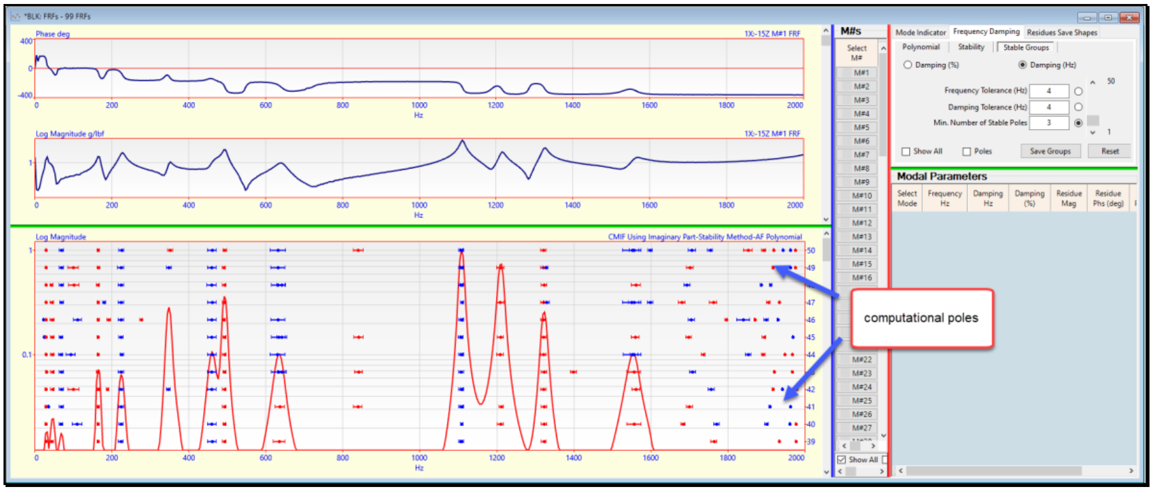
Only **Stable Pole Groups** are displayed on the **Stability** diagram, unless **Display All** is checked.

A **Stable Pole Group** must meet ***all the following criteria*** listed on the **Stable Groups** tab

- All poles in a Stable Pole Group ***must have*** **frequency estimates within the Frequency Tolerance**
- All poles in a Stable Pole Group ***must have*** **damping estimates within the Damping Tolerance**
- A Stable Pole Group ***must contain a*** **Min. Number of Stable Poles** that satisfy the Frequency & Damping Tolerances

Stable Groups will change if the criteria for defining a Stable Group are changed.

- If the **Min. Number of Stable Poles** ***is too small***, computational Poles will be displayed on the edges of the **Stability** diagram, as shown below

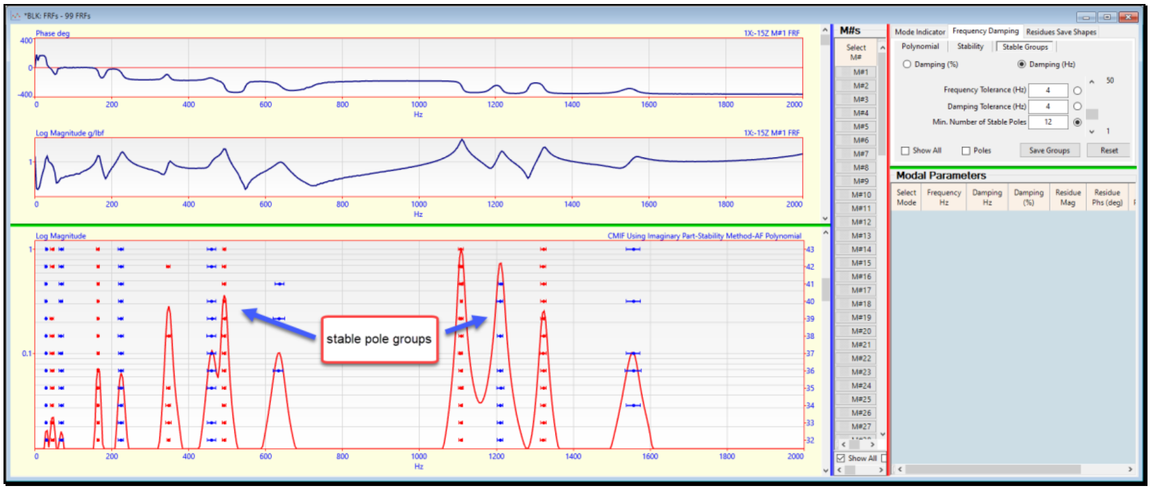


Computational Poles for ***Min. Number of Stable Poles → 4.***

Stable Group Colors

All poles in a Stable Group are displayed ***using the same color*** on the **Stability** or Poles diagram

Stable Group Colors alternate between the ***top two Contour Colors*** chosen in the **File | Data Block Options** box.

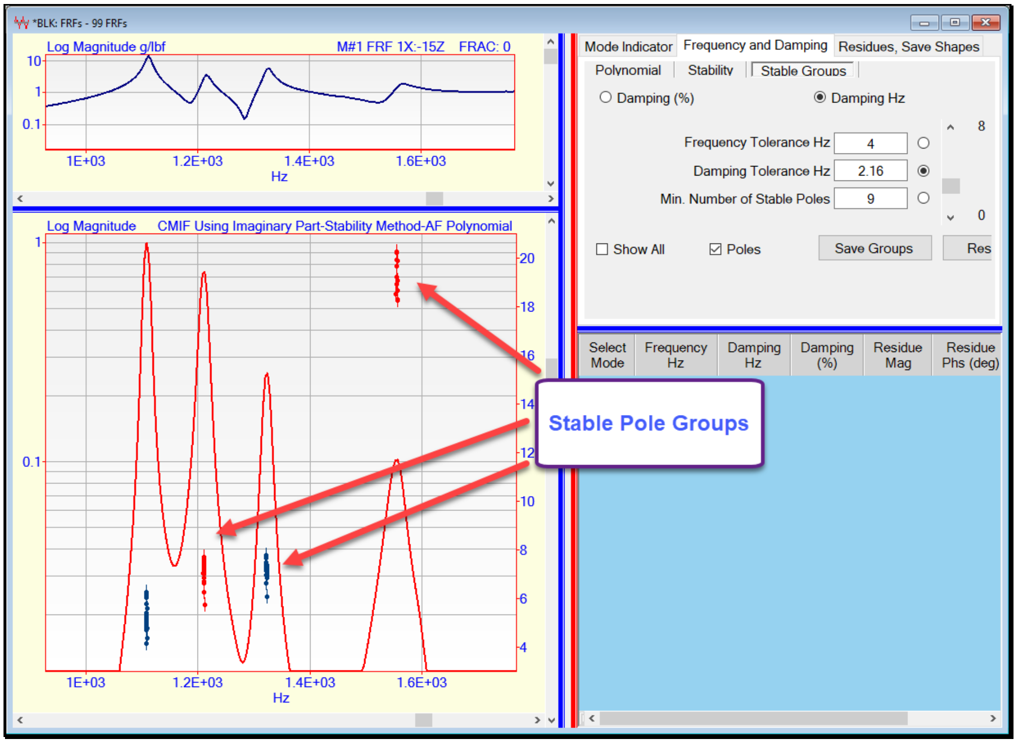


Stability *diagram Showing Stable Pole Groups.*

Poles Diagram

When the **Poles** box is ***checked*** on the Stable Groups tab, Poles estimates are displayed on a **Poles** diagram as shown below

- **Modal frequency** estimates are plotted along the ***horizontal axis***
- **Modal damping** estimates are plotted along the ***vertical axis***



Poles Diagram Showing Stable Groups

Changing the Stable Groups Settings

The **Stability** diagram is updated whenever the Frequency Tolerance, Damping Tolerance, or Min. Number of Stable Poles settings on the Stable Groups tab is changed.

To change the Frequency Tolerance, Damping Tolerance, or Min. Number of Stable Poles,

- ***Click*** on a radio button and ***scroll the slider*** on the ***right side***
- Or type a number into the box next to the setting
- ***Press*** the **Reset** button to reset the **Stability** settings to default values

Displaying Damping Values

Damping values can be displayed on the **Stability** diagram either as a **percentage of critical damping (%)** or as the **3 dB** or **half power point damping (in Hz)**.

- Select **Damping (%)** or **Damping (Hz)** on the Stable Groups tab to display damping in those units

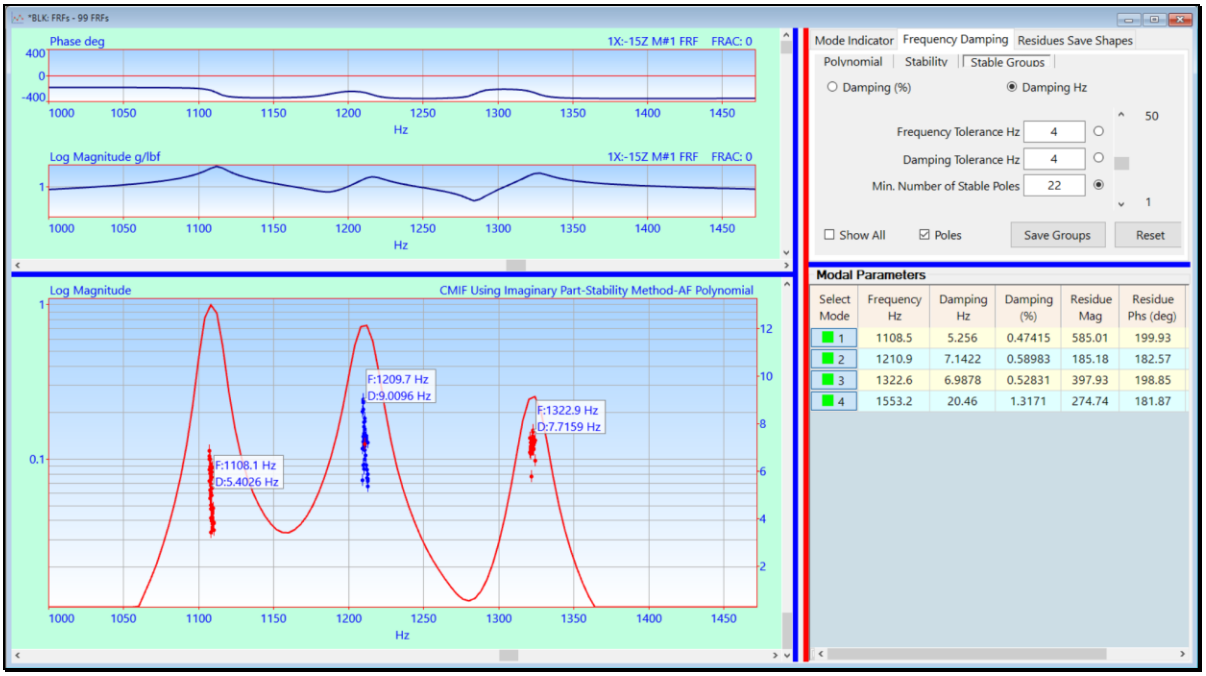
Displaying Pole Values

To display a pole value on the **Stability** or **Poles** diagram

- Place the mouse pointer ***near a pole*** and ***left click*** near a pole to display its pole values

To remove the display of a pole value,

- Place the mouse pointer ***on the pole value*** and ***left click***



Pole Values Displayed on Poles Diagram

Curve Fit | Stability | Clear Stability

Clears the poles from the **Stability** diagram.

Curve Fit | Stability | Stability Diagram (Stability button)

Creates a **Stability** diagram by curve fitting ***all*** or ***selected*** **M#s** in a Data Block (**BLK**).

Curve Fit | Stability | Stability Reset (Reset button)

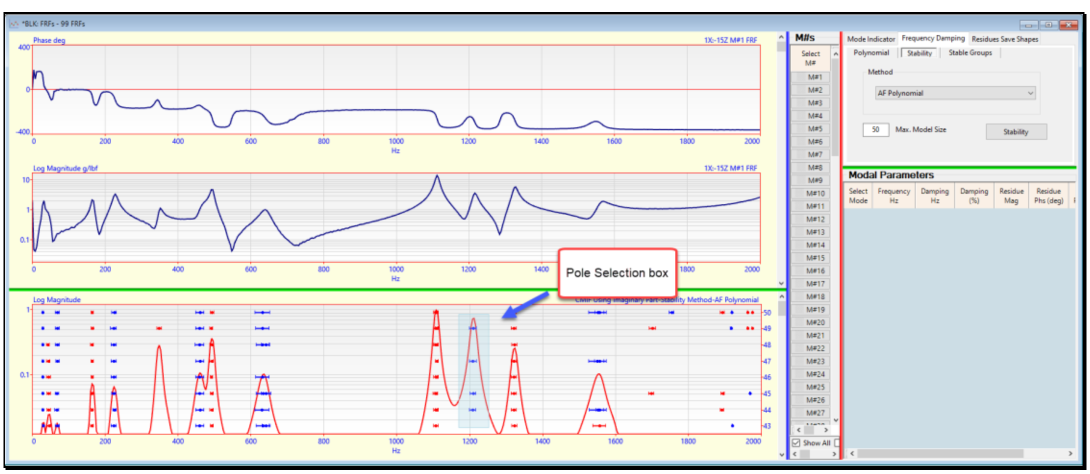
Resets the Stable Group parameters to default values.

Curve Fit | Stability | Poles Selection Box

Enables the Poles selection box on the **Stability** diagram.

When a Pole Selection box is drawn, the ***average value*** of the poles in the selection box is added to the **Modal Parameters** spreadsheet.

- **Click & drag** on the **Stability** or **Poles** diagram, to draw a selection box and enclose one or more poles



Pole Selection Box Drawn on the ***Stability*** *Diagram.*

Curve Fit | Stability | Save Stable Groups (Save Groups button)

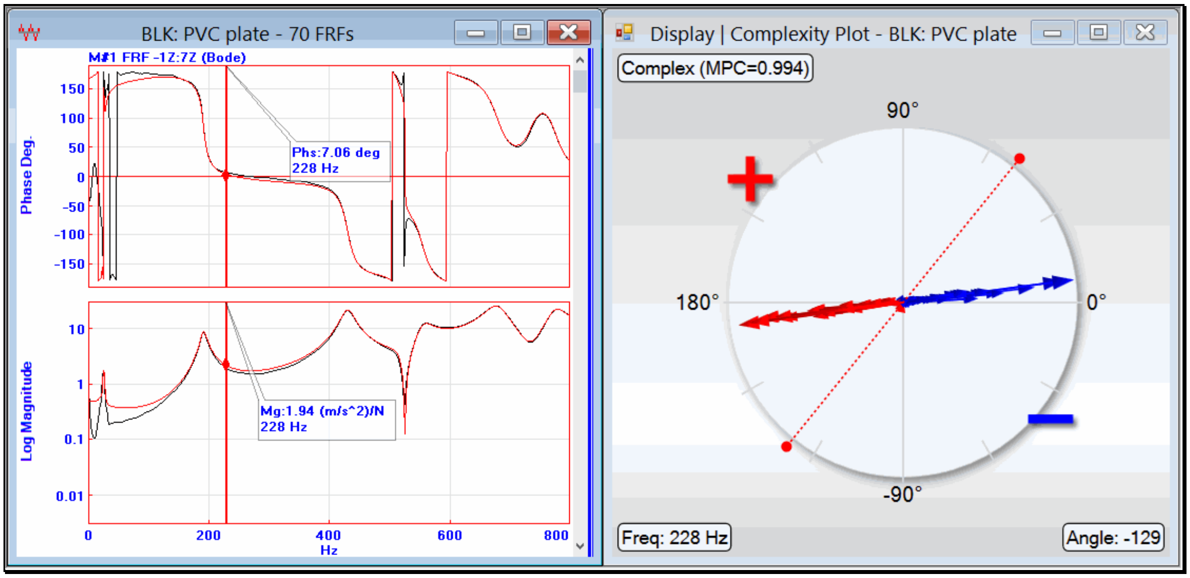
When this command is executed (or the **Save Groups button is *pressed***), the ***average value of all poles*** in each Stable Pole Group is added to the **Modal Parameters** spreadsheet.

- If the **Band** cursor is displayed, the ***average value*** of each Stable Pole Group ***within the cursor band*** is added to the **Modal Parameters** spreadsheet

Display | Complexity Plot Data Block (BLK)

Opens the **Complexity Plot** window from a Data Block (**BLK**) window.

A Complexity Plot displays the **magnitudes** & **phase *all*** (or ***selected***) **M#** values at the ***current Cursor position***



*Data Block (**BLK**) Complexity Plot.*

Normalized Mode Shape

Each shape component of a **normalized mode shape** has a phase of **0** or **180 degrees**.

A normalized mode shape is also called a ***normal mode shape***.

An FEA model with no damping yields ***normal mode shapes***.

During shape animation, a normalized shape exhibits a ***standing wave motion***, and its **node lines *do not move.***

The shape components of a normalized shape ***lie on a straight line*** in a Complexity Plot.

Complex Mode Shape

Each shape component of a **complex shape** can have an **arbitrary phase**.

During shape animation, a **complex shape** can exhibit a ***traveling wave*** motion, and its **node lines *can move***.

The shape components of a complex shape ***do not lie on a straight line*** in a Complexity Plot.

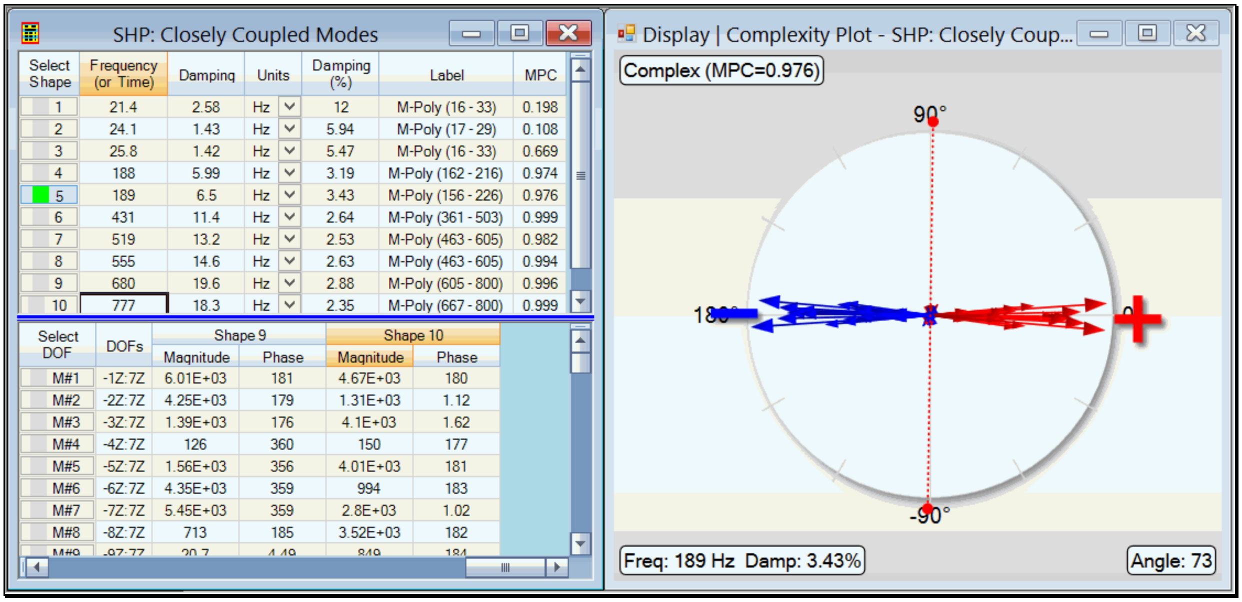
Experimental mode shapes can be ***complex*** for several reasons

- Real-world structures with ***heavy damping*** will have complex mode shapes
- ***Measurement errors*** can introduce ***arbitrary phase angles*** into EMA mode shape estimates
- ***Curve fitting errors*** can introduce ***arbitrary phases angles*** into EMA mode shape estimates

Display | Complexity Plot Shape (SHP) Table

Opens the **Complexity Plot** window from a **Shape (SHP) Table** window.

The Complexity Plot displays the magnitudes & phases for ***all (or selected)*** shapes for ***all*** **(or *selected*) M#s**



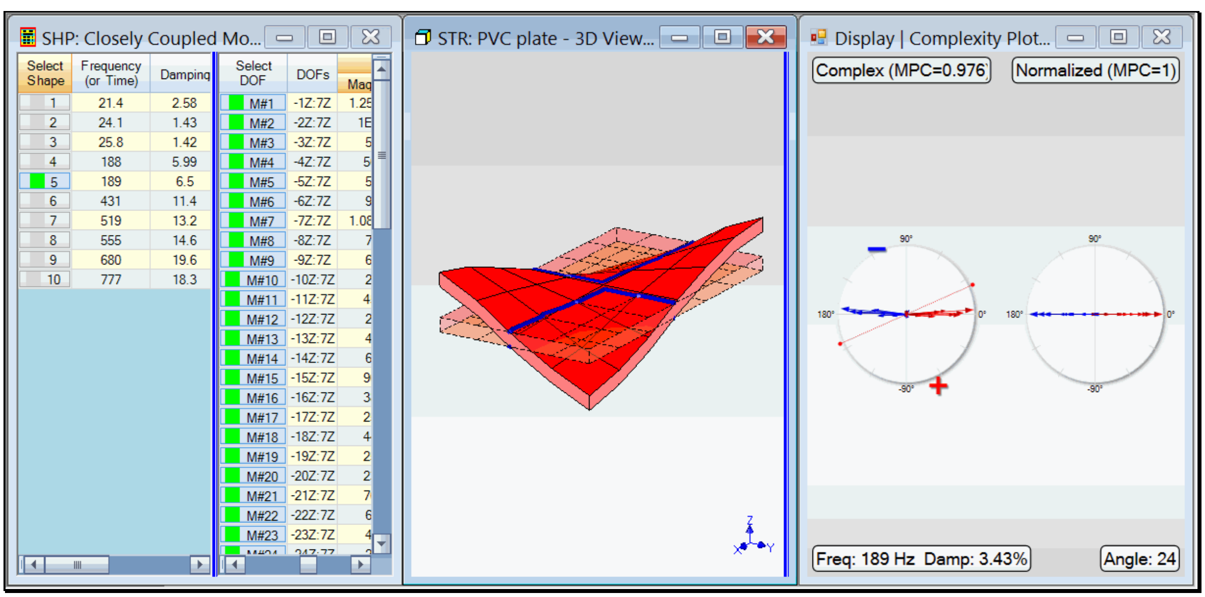
Shape (SHP) Table *Complexity Plot.*

Normalized Shapes on the Complexity Plot

When a complex shape is ***normalized***, it behaves like a normal mode shape with real valued shape components.

When a complex shape is ***normalized***, each shape component magnitude is retained, but its phase is changed to either **0** or **180** degrees

When **Display | Normalized Shape** is ***checked***, ***complex*** shape components are displayed on the ***left***, and ***normalized* shape components** are displayed on the ***right*** of the Complexity Plot, as shown below.



Complexity Plot with Display | Normalization ***Checked****.*

Normalization Line

When a shape is ***normalized***, the **red (+)** shape components are given **0 degrees phase**, and the **blue (-)** shape components are given **180 degrees phase**

The **normalization line** (***dashed line)*** on a Complexity Plot is used to ***normalize*** each complex shape.

- Each shape in a **Shape (SHP) Table** can have a different normalization line

To rotate the normalization line to a different angle

- ***Click & drag near*** **the normalization line** on the Complexity Plot

Flipping the Phases of the Right-Hand Shape

During **Animate | Animate a Pair**, if two similar shapes are animating 180 degrees out of phase with one another, the phases of ***right-hand*** shape can be changed by 180 degrees so that the two shapes animate more closely together.

- Execute **Animate | Animate a Pair | Flip Right Sign** in the Structure (**STR**) window to multiply the ***right-hand shape by "-1"***
- Or ***drag & rotate the the normalization line*** on the Complexity Plot to flip the phase of the shape components

Modal Phase Co-linearity (MPC)

MPC is a measure of whether or not the components of a complex shape ***are normalized***. they ***lie on a straight line***.

MPC has values between 0 &1.

- If **MPC** = **1** ➔ all components on the shape ***lie on a straight line***
- If **MPC < 1** ➔ some shape components ***do not lie on a straight line***
- If **MPC is *"close to 1"*** ➔ the structure ***is lightly damped***, or the shape is a ***normalized shape***

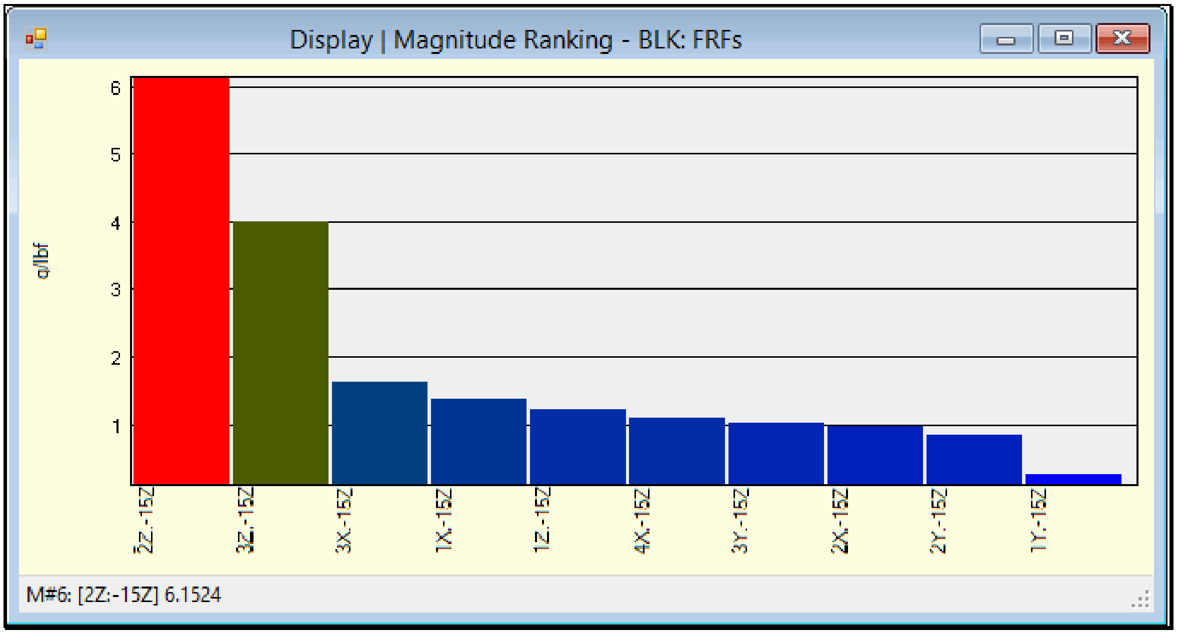
Display | Magnitude Ranking Data Block (BLK)

Opens the Magnitude Ranking window from a Data Block (**BLK**) window.

- **M#** magnitudes are plotted on the ***vertical axis*** versus (DOFs or **M#s**) on the ***horizontal axis***
- The magnitudes are ranked from the ***largest on the left*** to the ***smallest on the right***
- The **Contour Colors** defined in the **File | Data Block Option**s box are used for the Bar colors

M# magnitudes of the **M#s** at the ***current cursor position*** are displayed

- If ***no cursors*** are displayed, magnitudes of ***all M#s*** are displayed
- Magnitude Ranking ***is updated when the cursor is moved***, or ***different* M#s *are selected***



*Data Block (**BLK**) Magnitude Ranking Chart.*

Which Magnitudes Are Ranked?

- If the **Real** part of the **M#s** is displayed, the **Real** parts are ranked
- If the **Imaginary** part of the **M#s** is displayed, the **Imaginary** parts are ranked
- Otherwise**,** the **Magnitudes** of the **M#s** are ranked

Display the Magnitude Value

- ***Hover*** the mouse pointer over a **magnitude bar** to display its value at the bottom of the window

Display | Magnitude Ranking Shape (SHP) Table

Displays the magnitudes of the **M#s** for ***all*** (or ***selected***) **shapes** and ***all*** (or ***selected***) **M#s** in a **Shape (SHP) Table**.

- Magnitude Ranking ***is updated*** when ***different shapes are selected***, or ***different* M#s *are selected***

Tools | Modal Decomposition

Decomposes time or frequency **M#s** in a Data Block (**BLK**) into ***"resonance curves"*** that represent the contribution of each mode shape at each sample of **M#** data.

This command uses the same equations as the **Tools | Modal Participation** command in a **Shape (SHP) Table** but ***applies them at each sample*** of **M#** data.

Only mode shapes are used for decomposition. Modal frequency & damping are not used.

The following equation is solved at ***each sample of time or frequency*** data in the Data Block (**BLK**)

[Shapes] {Decomp(sample)} = {ODS(sample)}

(**n by m**) (**m by 1**) (**n by 1**)

[Shapes] ➔ matrix of mode shapes **(n by m)**. Each column of **[Shapes]** contains a mode shape

{Decomp(sample)} ➔ participation of each mode shape in the **{ODS}** at each sample

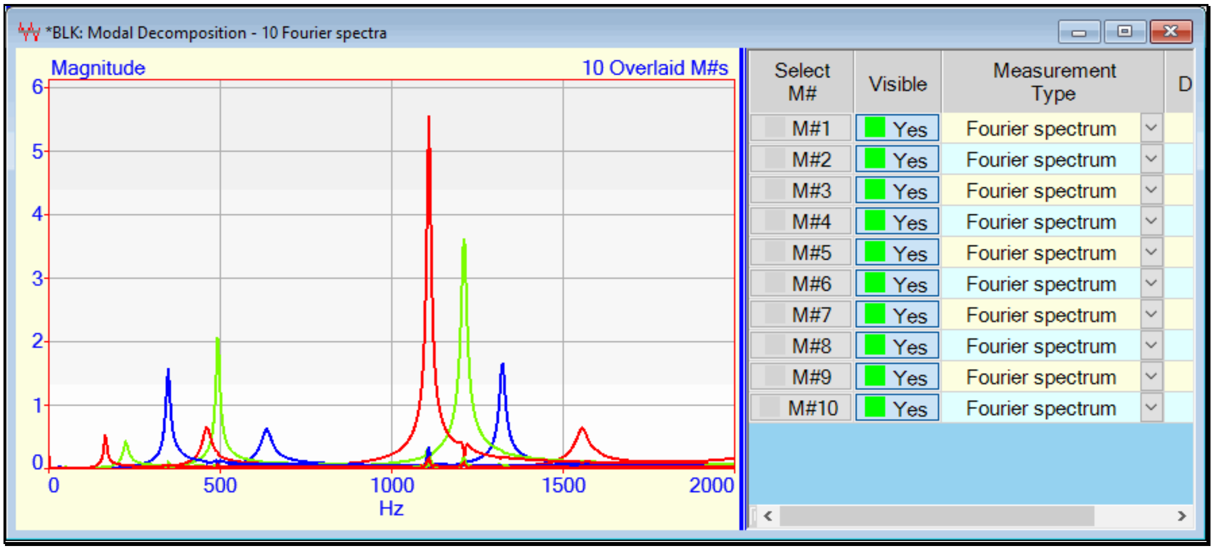
{ODS(sample)} ➔ **ODS** data ***at each sample*** in the Data Block (**BLK**)

n ➔ number of mode shape **DOFs**

m ➔ number of mode shapes

The mode shapes in **[Shapes]** must meet the following conditions,

- The mode shapes are valid for the structure ***regardless of boundary conditions***
- The mode shapes ***must have* DOFs *that match*** those of the Data Block (**BLK**)
- The mode shapes must be ***linearly independent*** of one another for the ***matching* DOFs**
- ***Linear independence*** of the mode shapes in **[Shapes]** can be validated by displaying their **MAC values** when the ***matching* DOFs *are selected***



***Modal Decomposition** into Ten Resonance Curves.*

Each resonance curve ***can be curve fit to extract the experimental modal frequency & damping*** associated with that resonance.

Tools | Modal Expansion

Expands the time or frequency **M#s** in a Data Block (**BLK**) using mode shapes.

Only mode shapes are used for **Modal Expansion**. Frequency & damping are not used.

This command uses the same equations as the **Tools | Modal Expansion** command in a **Shape (SHP) Table** but ***applies them at each sample*** of **M#** data,

The mode shapes must meet the following conditions,

- The mode shapes are valid for the structure ***regardless of boundary conditions***
- The mode shapes ***must have* DOFs *that match*** those of the Data Block (**BLK**)
- The mode shapes must be ***linearly independent*** of one another at the ***matching* DOFs**
- ***Linear independence*** of the mode shapes in **[Shapes]** can be validated by displaying their **MAC values** when the ***matching* DOFs *are selected***

Modal Expansion can be controlled by ***selecting* shapes** in the **Shape (SHP) Table** and/or ***selecting* M#s** in the Data Block (**BLK**)

The expanded **M#s** can be curve fit using any **FRF**-based curve fitting method.

Display | M#s | Effective Mass, Stiffness, Damping

Displays the effective mass, stiffness & damping (also called **generalized mass, stiffness & damping**) of each mode shape in the **M#s** spreadsheet.

> **Effective mass**, **stiffness** & **damping** are the values each mode would have if it were a single Mass-Spring-Damper located at a **DOF** of the mode shape.
>
> This command can only be used with **UMM** mode shapes

Effective mass, damping & stiffness are calculated for each mode with the formulas

Effective Mass = 1 / (Freq x Real (DP Residue) + Damp x Imaginary (DP Residue)

Effective Stiffness = (Freq^2 + Damp^2) x Effective Mass

Effective Damping = 2 x Damp x Effective Mass

Freq ➔ damped natural frequency of the mode

Damp ➔ half power point damping of the mode

DP Residue ➔ ***driving point Residue*** for each component of the mode shape

Tools | Modal Participation

Calculates the Modal Participation, the ***numerical participation*** of one set of shapes in another set of shapes.

> The Modal Participation equates the shapes in one **Shape (SHP) Table** with the shapes of another **Shape (SHP) Table**
>
> Each **Shape (SHP) Table** can contain **ODS**'s, EMA mode shapes, OMA mode shapes, FEA mode shapes or Engineering Data Shapes

Modal Participation Equation

If two sets of shapes are assembled into two matrices **[U] & [V]**, where ***each column*** of each matrix contains a shape, the two shape matrices are equated to each other with the following equation involving a Modal Participation.

[U] [Part] = [V]

[U] ➔ matrix of complex shapes (**m by n**)

[V] ➔ matrix of complex shapes (**m by q**)

[Part] ➔ complex Modal Participation (**n by q**)

n = number of shapes in **[U]**

q = number of shapes in **[V]**

m = number of matching DOFs between **[U] & [V]**

Writing out the matrices in terms of their components

$$\begin{bmatrix} u_{1,1} & \cdots & u_{1,n} \\ \vdots & \ddots & \vdots \\ u_{m,1} & \cdots & u_{m,n} \end{bmatrix} \begin{bmatrix} p_{1,1} & \cdots & p_{1,q} \\ \vdots & \ddots & \vdots \\ p_{n,1} & \cdots & p_{n,q} \end{bmatrix} = \begin{bmatrix} v_{1,1} & \cdots & v_{1,q} \\ \vdots & \ddots & \vdots \\ v_{m,1} & \cdots & v_{m,q} \end{bmatrix}$$

(m by n) (n by q) (m by q)

Participation as a Least-Squared-Error Solution

The Modal Participation is a ***least-squared-error solution*** to the above equation using the formula,

$$[Part] = [\,[U]^h[U]\,]^{-1}\,[U]^h[V]$$

h ➔ denotes the ***transposed conjugate matrix***

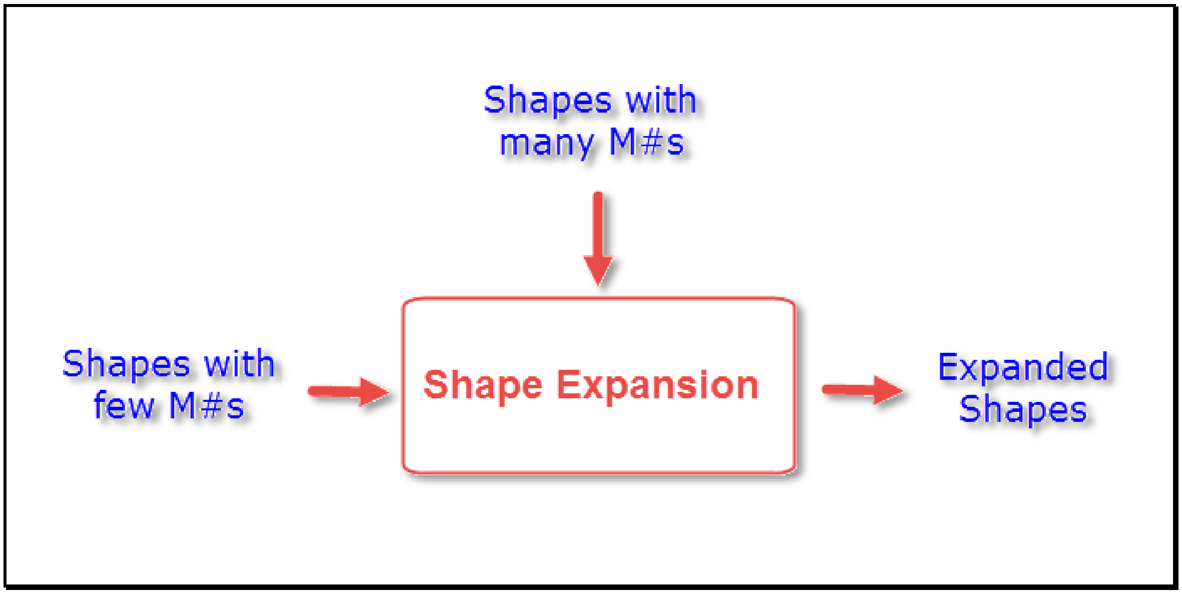
-1 ➔ denotes the ***inverse matrix***

- The shapes **[U]** must be valid for the structure ***regardless of boundary conditions***.
- Only the shapes in **[U]** are required for the Modal Participation calculation. Modal frequency & damping ***are not used***.
- The shapes in [U] & [V] must have ***some matching* DOFs**
- The shapes in **[U]** must be ***linearly independent*** of one another for the ***matching* DOFs** between the shapes in **[U] & [V]**
- ***Linear independence*** of the shapes in **[U]** can be validated by displaying their **MAC values** for the ***matching* DOFs**

Tools | Modal Expansion

Expands the number of DOFs in a **Shape (SHP) Table** using shapes ***with more* DOFs** from another **Shape (SHP) Table**.

- Modal Expansion calculates a **Modal Participation** as described under **Tools | Modal Participation**
- The **Modal Participation** is multiplied by the shapes ***with many* M#s** to obtain the ***expanded shapes***
- Any valid shapes for the structure can be used for Modal Expansion, ***regardless of their boundary conditions***
- Only mode shapes are used for Modal Expansion. Frequency & Damping are not required
- The frequency & damping of each un-expanded shape ***are retained with each expanded shape***



Equation for the Expanded Shapes

The matrix of expanded shapes **$[V_e]$** is calculated with the following equation,

$$[V_e] = \begin{bmatrix} V_m \\ V_u \end{bmatrix} = \begin{bmatrix} U_m \\ U_u \end{bmatrix} [Part]$$

$[U_m]$, $[V_m]$ ➔ sub-matrices with *matching* DOFs

$[U_u]$, $[V_u]$ ➔ sub-matrices with *un-matched* DOFs

[Part] ➔ Modal Participation

Modal Expansion can be controlled by ***selecting shapes*** and/or ***selecting*** **M#s** in either **Shape (SHP) Table**