

## Using Impact Testing for Production Quality Control

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### ABSTRACT

It is well known that when any physical property of a machine or structure changes, its modal parameters will also change. Modal frequency is the most sensitive parameter to changes in stiffness. If the stiffness of a test article is less than the stiffness of a properly assembled structure, its modal frequencies will be less than those of the properly assembled structure. If its stiffness is greater than the stiffness of a properly assembled structure, its modal frequencies will be greater than those of the properly assembled structure.

In this paper it is shown how a simple, low-cost *three-step* process can be used for pass-fail testing in a production environment, or for qualification testing of any assembled mechanical system. In the *first step*, the structure is impacted and Frequency Response Functions (FRFs) are calculated from the acquired force and response signals. In the *second step*, the FRFs are curve fit to obtain the modal frequencies of several modes of the structure. In the *third step*, the modal frequencies are *numerically compared* with the frequencies of a properly assembled structure.

A metric called the **Shape Difference Indicator (SDI)** is used to numerically compare two “*shapes*”. In this application, each “*shape*” contains the modal frequencies of several modes of the test article. A test article passes the qualification test when its SDI value with the shape of a correctly assembled article is “*close to 1*”.

In the example used for this paper, SDI is also used to search a database of modal frequency shapes. The search results are displayed in a *bar chart of SDI values* between the *current & archived* shapes of modal frequencies. Each shape in the database is associated with a *known amount of torque* applied to several cap screws that attach two aluminum plates together. When the modal frequencies of the test article fail to closely match those of a correctly assembled structure, the bar chart will also indicate how much torque must be applied to each cap screw to pass the test.

### INTRODUCTION

It is well known that the resonant vibration of any mechanical structure or system is closely correlated with its physical properties. In other words, if any physical property (mass, stiffness, damping) or its boundary condition changes, its resonant vibration will change to reflect the change in the physical property or boundary condition.

Each natural resonance is also called a mode of vibration. Each resonance is completely defined by its natural frequency, damping decay, and its distributed deflection shape. These parameters are commonly called the modal frequency, modal damping, and the mode shape of a resonance.

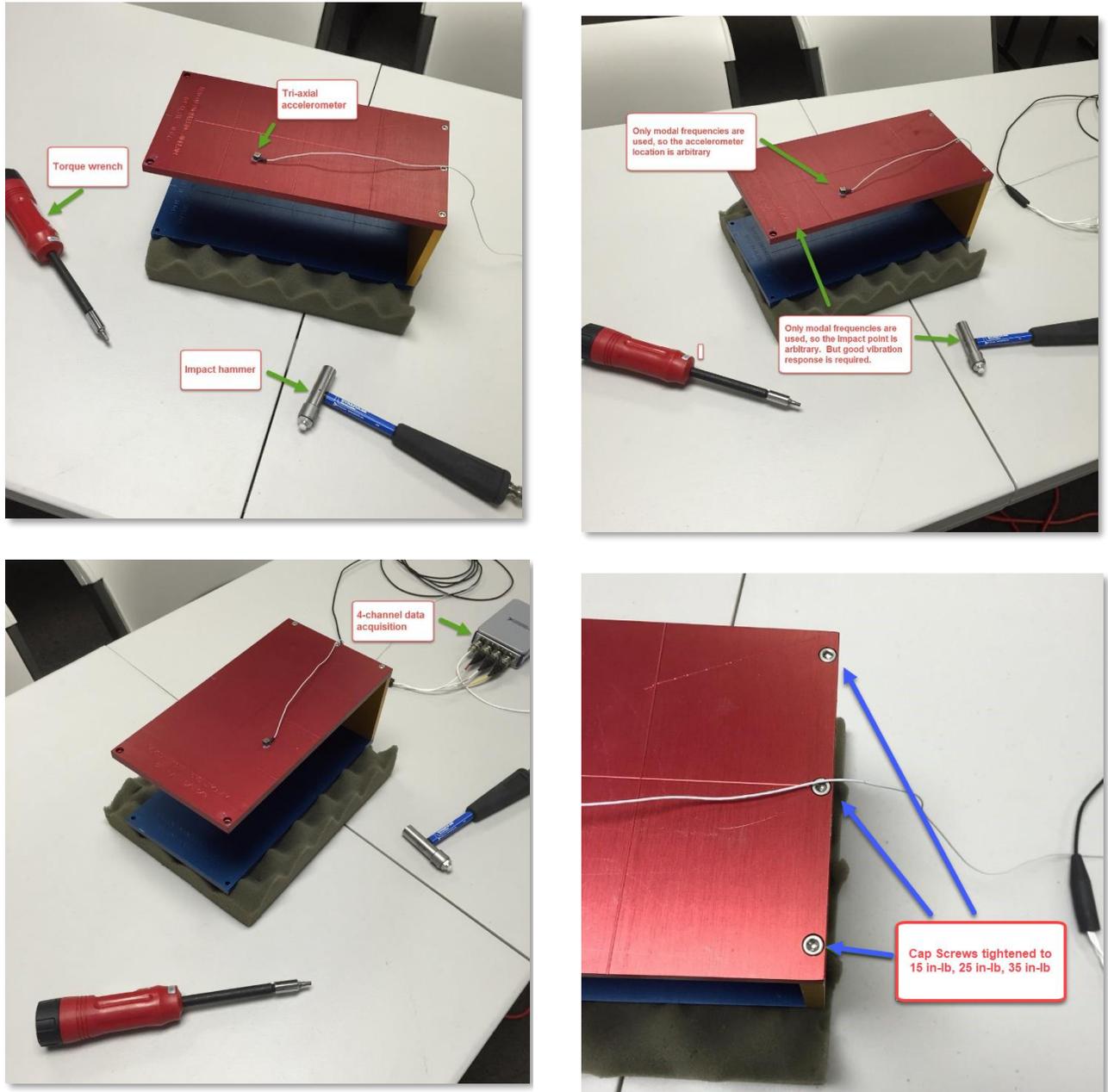
The frequency of a resonance is called its *natural frequency* because it is not dependent on the forces which cause the vibration. The same is true of modal damping and the mode shape. These three modal properties do not change unless a physical property or a boundary condition changes.

Modal frequency is the easiest parameter to estimate from experimental data. In a Frequency Response Function (FRF), each resonance is represented by a peak in the data. Modal frequencies are estimated by curve fitting experimental FRFs.

Modal frequency is the most sensitive parameter to stiffness changes in a structure. If stiffness is increased anywhere in a structure, its modal frequencies will increase. If stiffness is decreased anywhere in a structure, its modal frequencies will decrease.
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How much each modal frequency increases or decreases with a stiffness change depends on its mode shape and where the excitation forces are applied to the structure. If a force is applied where a mode shape has a non-zero

component, that mode will be excited and participate in the resonant vibration. Ideally, if its mode shape is zero where the force is applied, that mode will not be excited and will not participate in the vibration.



*Figure 1. Test Article*

In this example, different amounts of torque were applied to the three cap screws that attach the top plate to the back vertical plate of the Jim Beam structure shown in Figure 1. This has the effect of changing the joint stiffness between the two plates. If the joint stiffness changes, the frequencies of some modes will also change.

Different amounts of torque were applied to the three cap screws and the frequencies of several modes were determined by curve fitting experimental FRFs associated with each set of torques. The experimental modal frequencies associated with each set of torques were then used to determine whether the Jim Beam was properly assembled. One set of torques was assumed to be the values required for a correctly assembled beam structure.

## FaCTs™

The **Shape Difference Indicator (SDI)** metric was used to numerically compare two “*shapes*”, each containing modal frequencies corresponding to a different set of torques applied to the cap screws. SDI has values *between 0 & 1*. An SDI value *greater than 0.90* indicates a *strong correlation* between a pair of shapes. An SDI value *less than 0.90* indicates a *significant difference* between a pair of shapes.

SDI was used to search a database of archived modal frequency shapes; each shape associated with *known torque values* for the cap screws. Each cap screw was tightened using three different torque values, **15 in-lbs, 25 in-lbs, 35 in-lbs**. This gave a total of **27 different torque combinations** among the three cap screws.

Using SDI to search a database is called **Fault Correlation Tools** or **FaCTs™** [6]. With **FaCTs™**, the search results are displayed in a bar chart of the *ten highest SDI values* between the current & archived frequency shapes. The **FaCTs™** bar chart displays the ten SDI bars ordered from the highest to the lowest value. For this example, each bar was labeled with the three torque values associated with each frequency shape.

**FaCTs™** can be used in two different ways.

1. To pass or fail a test article based on its modal frequencies versus a *baseline* set of frequencies
2. To identify the correct amounts of torque necessary to pass a test article when it fails

## TEST SETUP

The Jim Beam was impact tested using a tri-axial accelerometer, an instrumented impact hammer, and a 4-channel acquisition system. The tri-axial accelerometer was attached to the top plate, and the impact force was applied to the top plate. FRFs between each of the three tri-axial responses and the impact force were then calculated. Finally, the three FRFs were curve fit to extract the modal frequencies of six modes of the plate. The six modal frequencies were then stored as “*shapes*” in an archival database.

This simple approach to qualification testing has several advantages,

1. Instead of an accelerometer, a non-contacting sensor such as a microphone or laser vibrometer could be used to measure the response
2. The impact force does not have to be measured. If it is not measured, Cross spectra between one of the responses and the others can be curve fit to obtain the modal frequencies
3. The location of the response sensor and the impact point can be different for each test
4. The only requirement for each test is that a resonance peak be present for each mode of interest in the FRFs or Cross spectra.

## Active Impact & Response Points

The location of the tri-axial accelerometer and the impact point are both arbitrary since modal frequencies can be extracted from practically any FRF. However, *active points* should be chosen so that resonance peaks of the modes of interest are non-zero in the FRFs or Cross spectra.

If the mode shapes of the modes of interest are available, one method of locating the most *active points* on the top plate is to multiply the shapes together and display their product on a model of the test article. The shape product of six modes of the Jim Beam is shown in Figure 2.

As shown in Figure 2A, the best locations for attaching the accelerometer and impacting the Jim Beam are on the two outer corners of the top plate. But if only the points in the middle of the top plate are considered, the best locations among those points are shown in Figure 2B. The active points shown in either Figure 2A or Figure 2B will provide valid measurements with resonance peaks for the six modes of interest in them.

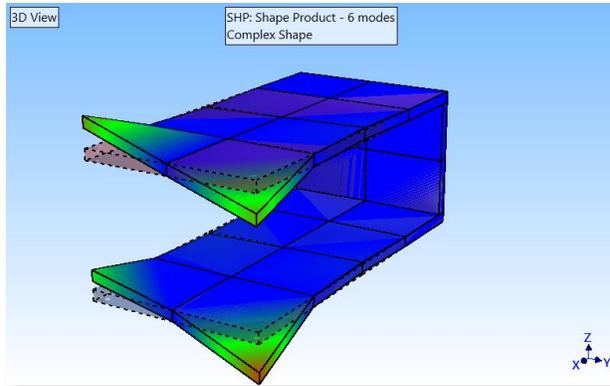


Figure 2A. Best Locations for Impact & Responses

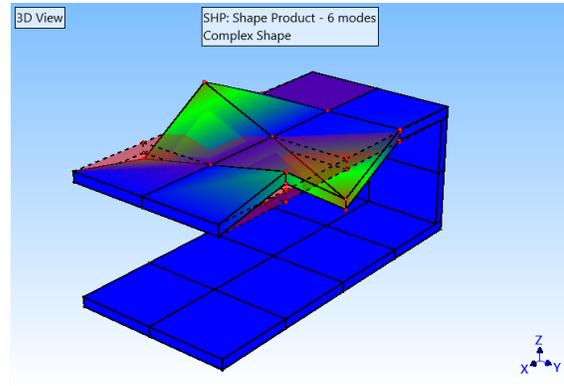


Figure 2B. Best Mid-Plate Locations

### MAC & SDI

The Modal Assurance Criterion (MAC) is a measure of the *co-linearity* of two shape vectors [1], [2]. If two shapes *lie on the same straight line*, they are *co-linear*, and *MAC equals 1.0*. If two shapes *do not lie on the same straight line*, they are *linearly independent*, and *MAC is less than 1.0*.

The Shape Different Indicator (SDI) is a measure of the *equality* of two shape vectors [3], [4]. If two shapes *have equal components*, *SDI equals 1.0*. If two shapes *do not have equal components*, *SDI is less than 1.0*. The following *rules of thumb* are used with SDI

- SDI values → *between 0.0 & 1.0*
- SDI = 1.0 → two shapes have *equal components*
- SDI > 0.9 → two shapes are *similar*
- SDI < 0.9 → two shapes are *different (some components are not equal)*

SDI can be used for comparing any pair of shape vectors that have numerical components. SDI has two advantages over MAC.

1. MAC measures the *co-linearity* of two shape vectors. Two shapes can be *unequal* but *still be co-linear*
2. MAC *cannot be used* to compare two shapes *having only one shape component*. MAC *equals 1* in those cases

### SDI Plot

In Figure 3, SDI is plotted between two shape vectors  $\{A\} = \{1, 1\}$  and  $\{B\} = \{x, y\}$ , where  $x$  &  $y$  have values for  $-5$  to  $+5$ . *SDI = 1* in only two places, where  $\{B\} = \{1, 1\}$  and  $\{B\} = \{-1, -1\}$ . When *SDI = 1*,  $\{A\}$  &  $\{B\}$  have *equal components*, but one can be the *negative* of the other.

Notice that *SDI = 0 at the origin*  $\{0, 0\}$ . Also, for values of  $\{A\}$  &  $\{B\}$  *near to the origin*, SDI *decreases rapidly toward zero*. This means that when  $\{A\}$  &  $\{B\}$  have components with *small numbers* in them, SDI is a *more sensitive* measure of their difference. Likewise, when  $\{A\}$  &  $\{B\}$  have components with *large numbers* (like modal frequencies), SDI is a *less sensitive* measure of their difference.

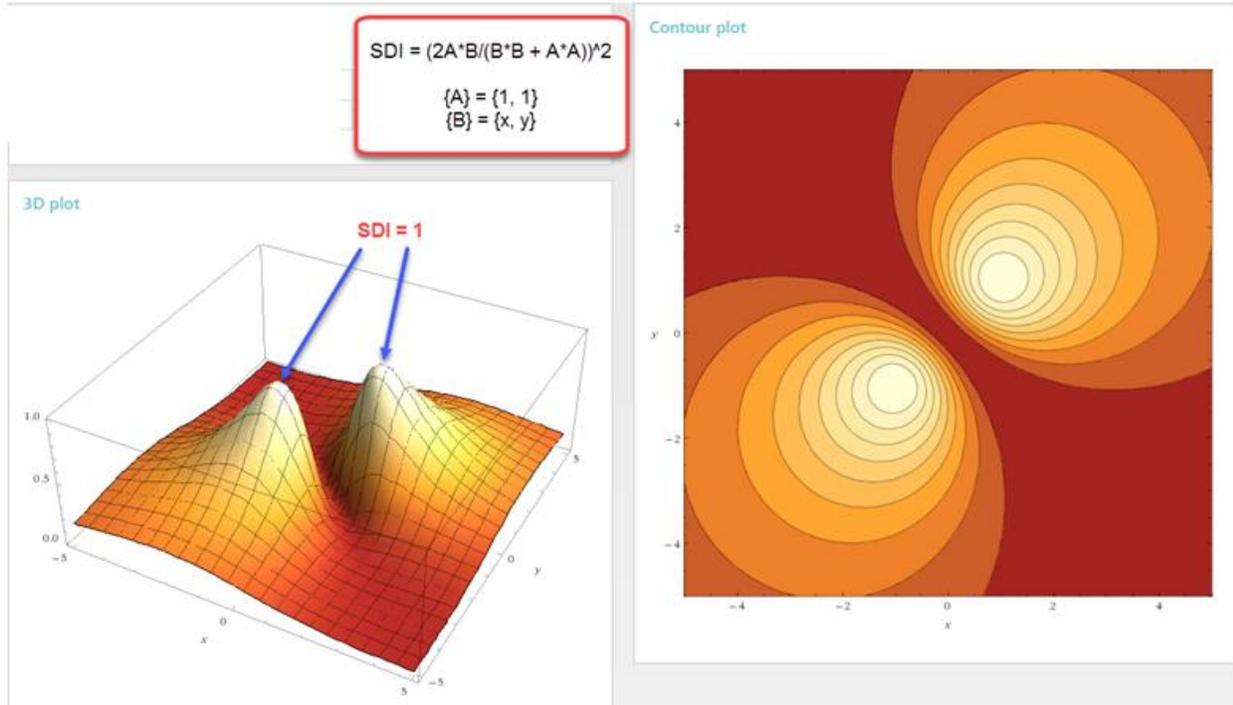


Figure 3. SDI for  $\{A\} = \{1, 1\}$   $\{B\} = \{x, y\}$ .  $x$  - -5 to +5,  $y$  - -5 to +5

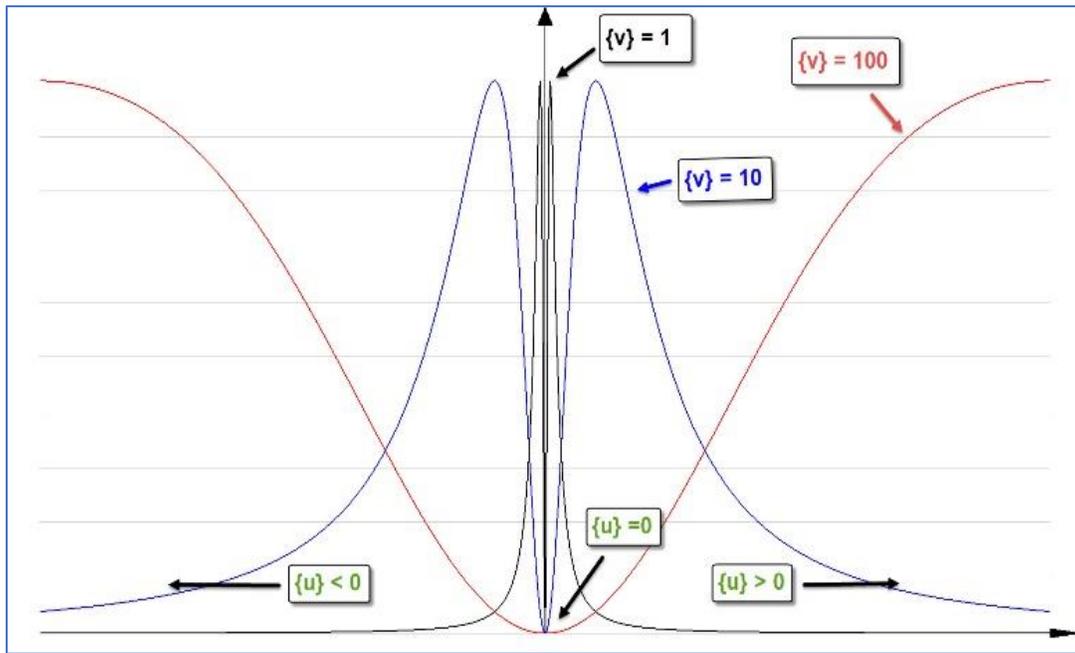


Figure 4. Sensitivity of SDI

### SDI Sensitivity

The sensitivity of SDI is more clearly shown in Figure 4. For large values of  $\{v\}$ , the SDI curve is “*very flat*”, meaning that it is less sensitive to the difference between  $\{u\}$  &  $\{v\}$ . For small values of  $\{v\}$ , SDI decreases rapidly when  $\{u\}$  &  $\{v\}$  are different. The FaCTs database search will more accurately distinguish between shapes with modal frequencies in them if SDI is made more sensitive by replacing the modal frequencies with small numbers.

## PASS-FAIL TEST

The following steps were used for Pass-Fail testing the Jim Beam

1. Impact the top plate and acquire the force signal and three response signals of the tri-axial accelerometer
2. Calculate three FRFs between the force & response signals
3. Curve fit the FRFs for the modal frequencies of six modes
4. Use FaCTs to compare the modal frequencies of the six modes with the *baseline frequencies* of a properly assembled structure
5. If FaCTs is *less than 0.90*, the structure *fails* the assembly test
6. If the structure *fails* the test, FaCTs identifies the cap screw torque values required to pass the test

## TORQUE CASES

The torque wrench shown in Figure 1 was used to tighten each cap screw using a known amount of torque.

- Three torque values were applied to each screw; **15 in-lbs, 25 in-lbs, 35 in-lbs**
- Applying **3 different torques** to each screw gave a total of **27 different torque cases**
- With one of the torques applied to each screw, the structure was impacted and three FRFs were calculated between the impact force & the responses of the tri-axial accelerometer
- A total of **81 FRFs** were calculated from the impact & response data
- The FRFs for each torque case were curve fit and **6 modal frequencies** were archived in a database for each case

## FRF Acquisition

FRFs were acquired with the 4-channel data acquisition hardware shown in Figure 1. A typical acquisition of time waveforms and their resulting FRFs & Coherences is shown in Figure 5.

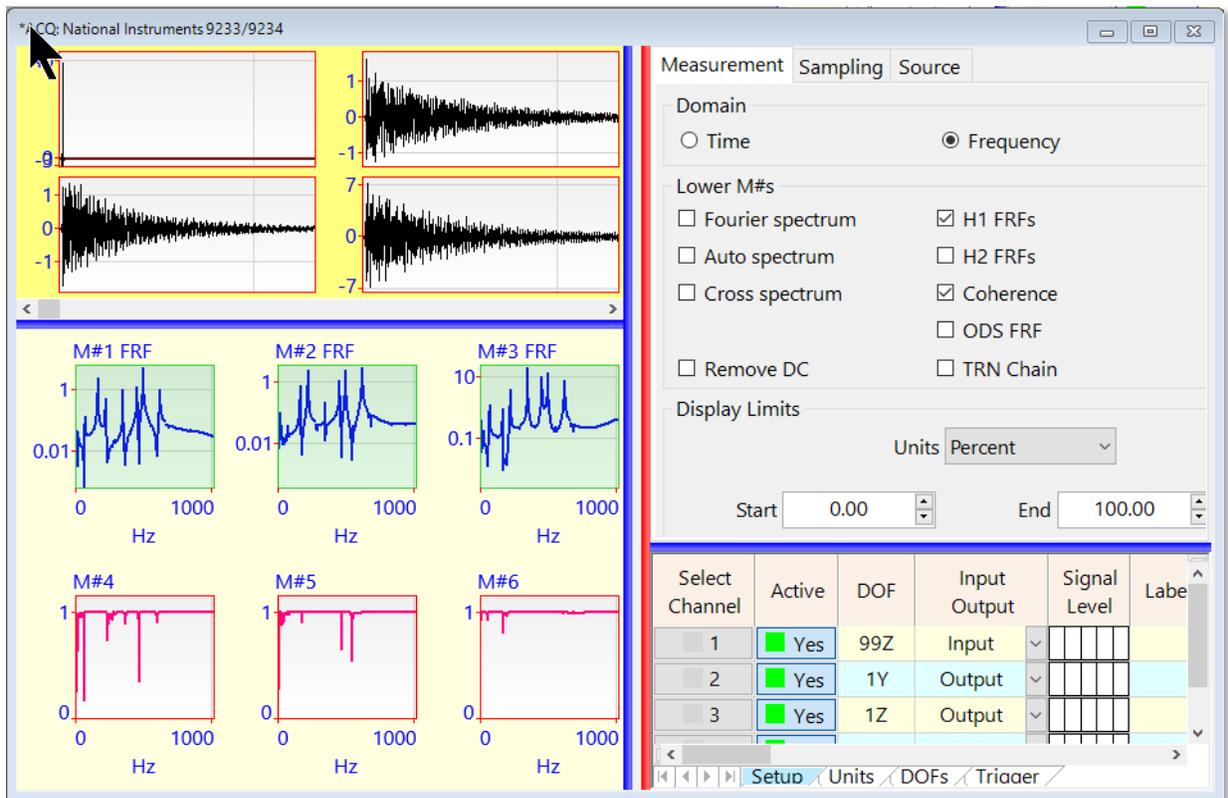


Figure 5 Time Waveforms, FRFs & Coherences

## TREND PLOT AND FaCTs BARS

A Trend Plot of the all 27 torque cases is shown on the left in Figure 6. Assuming the correct torque for each cap screw is **25 in-lbs**, the FaCTs bar chart on the right in Figure 6 shows the 10 torque cases with the highest FaCTs bars compared to the **25-25-25 case**.

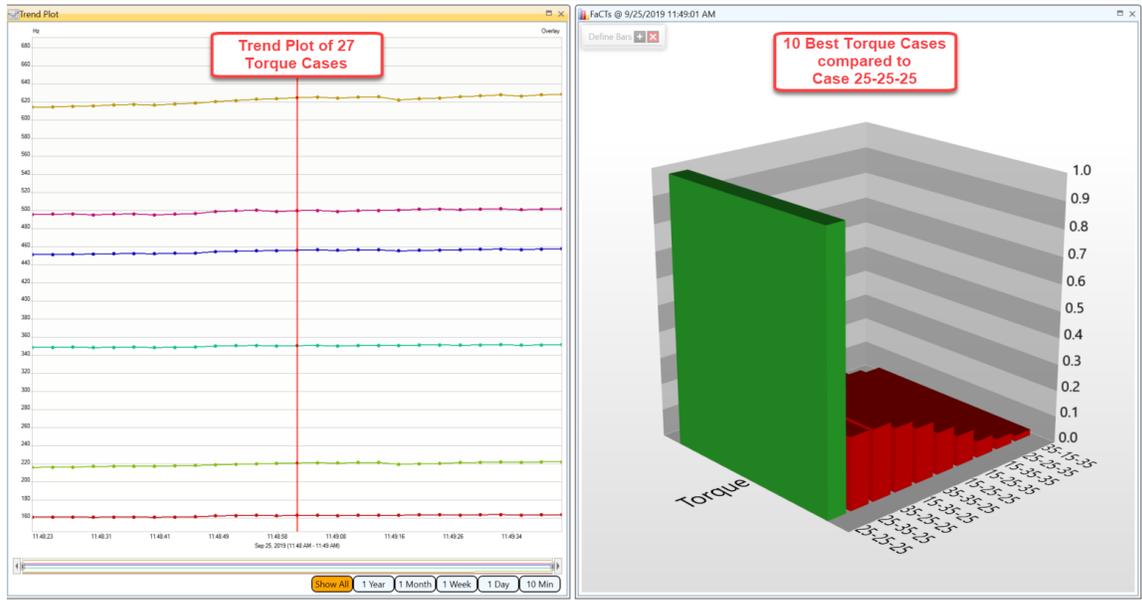


Figure 6 Trend Plot and FaCTs Bars

The FaCTs bar for the **25-25-25 case** has a value of “1”. The remaining FaCTs bars are ordered from highest to lowest according to their SDI values, and each FaCTs bar is labeled with its three torque values.

The 10 FaCTs bars show how the torque of each screw must be changed in order to match the **25-25-25 case** and pass the test article. The torque corrections for the five highest FaCTs bars are,

- 25-35-25 case → less torque on the middle screw
- 35-25-25 case → less torque on the first screw
- 15-35-25 case → more torque on the first screw & less torque on the middle screw
- 35-35-25 case → less torque on the first & middle screws
- 15-25-25 case → more torque on the first screw

Anyone of these torque corrections would pass the test article in this simulated production test.

### What About Sensitivity?

The modal frequencies have values between 160 Hz & 630 Hz. The SDI sensitivity graphs in Figure 4 indicate that the SDI curve is “*very flat*” for these large numbers. Hence, FaCTs will not be very sensitive to small differences in these modal frequencies. In a previous paper [3], a formula was given for replacing large numbers with small numbers in the shapes so that they are closer to the origin where SDI is more sensitive for measuring the difference between two shapes.

FaCTs uses sensitivities between 0 & 10. For sensitivity = 0, the shape components are not changed, and hence SDI is not very sensitive to the frequency differences. For sensitivity = 1 to 10, the shape components are replaced with smaller numbers.

Figure 7 shows FaCTs bar charts for the two extreme cases; **sensitivity = 0** and **sensitivity = 10**. In both cases, the correct torque case **25-25-25** case was still identified, having a FaCTs value of “1”. But when the maximum sensitivity is used, the FaCTs bars clearly distinguish the torque case of the properly assembled structure from the other torque cases.

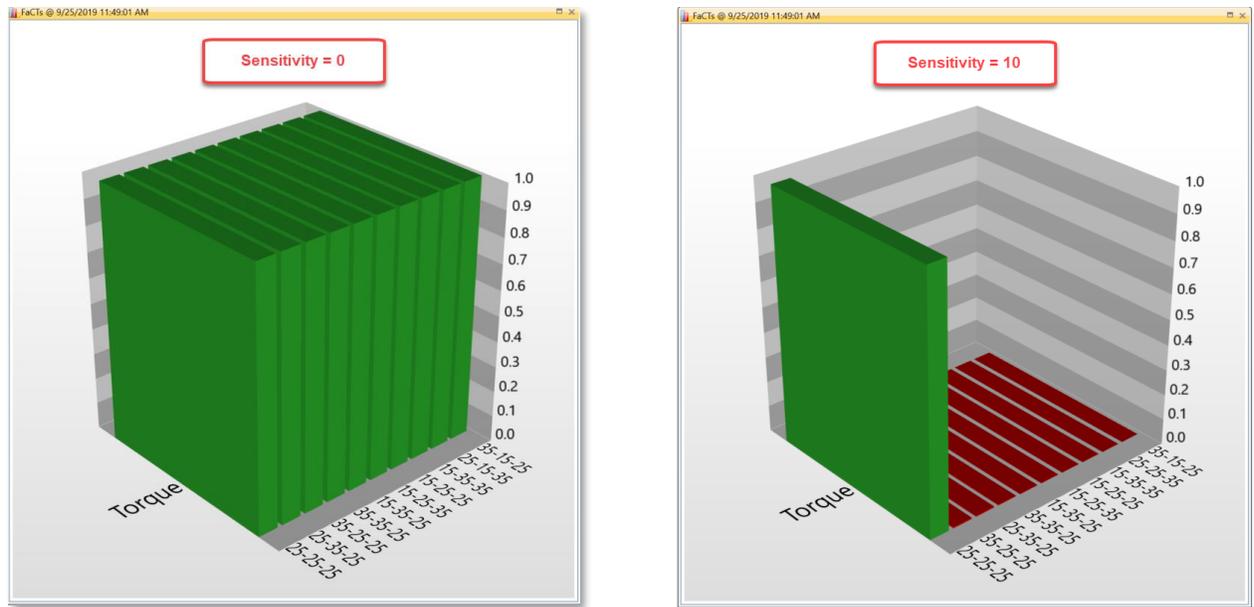


Figure 7 FaCTs Bars (Sensitivity = 0 & Sensitivity = 10)

## CONCLUSION

In the example presented here, the three cap screws used to attach two plates of the Jim Beam together were tightened using a torque wrench. Three different amounts of torque were applied to each screw, using a *very small difference on only 10 in-lbs* between the different torques.

SDI was introduced as a new metric for comparing two sets of shape data in reference [3]. It was later used as part of the database search method in reference [5] to identify unbalance in rotating machinery, and in reference [4] to identify cap screw torque values of a single cap screw from differences in modal frequencies. This paper extended the application in [4] by applying torque changes to three cap screws instead of just one.

SDI has two properties that make it useful for comparing two sets of shapes,

1. When SDI *equals 1* two shapes are either equal, or one is the negative of the other
2. When SDI is *less than 1*, the two shapes are not equal
3. SDI can be applied to shapes with *only one component*

SDI was used to search a database of shapes with modal frequencies in them. Each shape was associated with torques applied to the three cap screws used to attach the top and vertical plates of the Jim Beam structure together. The search method, called **FaCTs™**, displays a bar chart with up to the *ten highest SDI values* between the current shape and shapes archived in a database.

FaCTs identified the correct torque case from among 27 different cases, even with very small differences (10 in-lbs) between torque values. This confirmed that,

***Experimentally determined modal frequencies can be used in a product test or qualification test to identify very small stiffness changes in an assembled structure.***

Not only could FaCTs identify the correct torque case, but by ordering the 9 other cases according to their SDI values, FaCTs provided the torques necessary to pass a structure that initially fails a qualification test.

It was also shown that when two sets of shape data contain large numbers, as is the case with modal frequencies, SDI, and therefore FaCTs, can be made more sensitive to the difference between two shapes by replacing large shape values with smaller values.

The simplest use of SDI as a Pass-Fail indicator is to compare each current set of shape data with a *baseline* set of data acquired from a correctly assembled machine or structure. When SDI is *greater than 0.9*, the test article is correctly assembled and passes the qualification test. When SDI is *less than 0.9*, the difference between the *current & baseline* shapes is *significant* and the test article has failed the test.

FaCTs can be used in a wide variety of production testing or machine qualification testing environments since it provides an accurate way of comparing multiple sets of shape data. The bars in a FaCTs bar chart are easy to understand since they only have values between 0 & 1.

## REFERENCES

- [1] - R. J. Allemang, D. L. Brown "A Correlation Coefficient for Modal Vector Analysis", Proceedings of the International Modal Analysis Conference 1982
- [2] - R. J. Allemang "The Modal Assurance Criterion (MAC): Twenty Years of Use and Abuse", Proceedings of the International Modal Analysis Conference, 2002
- [3] - S. Richardson, J. Tyler, P. McHargue, M. Richardson, "A New Measure of Shape Difference", IMAC XXXII, February 3-6, 2014
- [4] - S. Richardson, J. Tyler, B. Schwarz, P. McHargue, M. Richardson "Using Modal Parameters for Structural Health Monitoring" IMAC-XXXV January 30–February 2, 2017 Garden Grove, CA
- [5] - S. Richardson, M. Richardson, J. Tyler, P. McHargue, "Using Operating Data to Locate & Quantify Unbalance in Rotating Machinery", IMAC XXXIV, January 25-28, 2016
- [6] - **FaCTs™** is a trademark of Vibrant Technology, Inc.